USING THE FUNDAMENTAL DIAGRAM TO MONITOR TRAVEL TIMES AND MOTORWAY PERFORMANCE: METHODOLOGY AND CASE STUDY

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ABSTRACT

Motorway owners and operators are now using improvements in travel time reliability (a reduction in travel time variability) as an added benefit of motorway use and a key performance indicator (KPI).This paper introduces a novel use of the Fundamental Diagram (FD) to monitor travel times and performance on a motorway. The methodology is tested in a case study in collaboration with Leighton Contractor's on a new motorway. KPIs on travel times are to be used to financially incentivise the operator to maintain a high level of travel time reliability. The KPI definitions that were initially proposed were based on criteria that would have led to penalties being applied even under normal operation. Hence a more robust and equitable approach to KPI calculation was required.

We report on the design and development of such an approach, which has been accepted by all stakeholders in the project. It is based on the Fundamental Diagram, which is used to define a base line for equilibrium traffic conditions. Calculation of the Fundamental Diagram is based on a recently proposed model. We then construct an envelope around the FD. The boundaries of the envelope signify normal or "healthy" traffic conditions. Travel time KPIs penalties are only triggered for traffic flow data points that are outside the agreed "healthy" flow envelope.

INTRODUCTION

Traditionally, a road agency might value the benefit of a project by virtue of the fact that the project was delivered as designed and therefore it is inferred that it has delivered the benefit that it promised in the economic business case. With the proliferation of the use of private funding models to infrastructure procurement, one aspect being increasingly desired is to procure outcomes that align to the benefit that the project is intended to provide. In other words, the measure of success is that the investment has paid off at the strategic level. Such measures might include improvements in travel time, reductions in road trauma, improved asset availability or even stimulus of economic activity. With that approach taken to procurement, it requires the contract to be anchored in performance-based KPIs that are determined prior to contract execution, long before the road is designed, let alone constructed.

In this case the issue was to develop a robust and equitable method for setting a Travel Time KPI on a new motorway due to start operating in 2020.

The problem was how to position a travel time KPI that best delivers client and contractor outcomes without requiring a significant risk premium being priced into the contractor's bid. Specifically the contractor needed an approach that will use the highly detailed traffic flow data that will be available to administer a KPI that provides the motorway owner assurance that the contractor is incentivised to protect travel time reliability. The KPI definitions that were initially proposed were based on criteria that would have led to penalties being applied even under normal operation. Hence a more robust and equitable approach to KPI calculation was required.

In collaboration with Leighton Contractors, a methodology for setting the KPI was developed, which has been accepted by all stakeholders in the project. It is based on the Fundamental Diagram, which is used to define a base line for equilibrium traffic conditions. The core of the

approach is to construct an envelope around the Fundamental Diagram curve which signifies normal traffic conditions. Travel time KPIs penalties are only triggered for traffic flow data which falls below the FD envelope and is an indicator of abnormal traffic conditions.

From a practical point of view, an operational system to assess travel time performance using the Fundamental Diagram needs the following:

1. A methodology for fitting the Fundamental Diagram curve from traffic flow data

2. A methodology for deriving critical density

3. A transparent methodology for setting bands or "envelopes" that distinguish between satisfactory and non-satisfactory travel times

4. How the methodology will estimate the envelope where there are insufficient data points above critical density

We present the proposed methodology of using FD for KPI calculation which satisfies all the four points above. The paper is structured as follows: In section 2, we review the background on the Fundamental Diagram (FD) as well as a detailed example on estimating an FD flow-density model applied to data from the a motorway in Brisbane, which is expected to be similar to the new motorway in terms of traffic parameters. Section 3 has a thorough discussion of the methodology for setting envelopes around the FD curve applied to this sample data. Finally, a conclusion is given in Section 4.

THE FUNDAMENTAL DIAGRAM AND "HEALTH" OF TRAFFIC FLOW: THEORETICAL AND EMPIRICAL BASIS

Since the 1930's, the Fundamental Diagram (FD) has been used as the description of the relationship between flow rate and density in steady state for uninterrupted traffic flow on a motorway. This type of FD is called flow-density FD. From the relationship:

Flow = density * space mean speed

the flow-density FD can be represented as a speed-density or flow-speed FD in a straightforward way. The existence of the Fundamental Diagram (FD) and the relationship between speed and density (or equivalently, flow and density or speed and flow) has been supported in hundreds of traffic observations and studies around the world in the last 70 years. The FD has become *the fundamental law of traffic flow.*

Historically, the concept of FD was prosed from experimental observations by US traffic engineer Bruce Greenshields in 1933, who postulated a linear relationship between speed and traffic density. However it has long been observed that a linear specification often describes the data poorly. For example, quite often the FD has a crucial point at which the traffic changes from free-flow to congested condition and correspondingly the functional relationship changes.

Various other functional relationships between the speed V and density k have been proposed to account for the observable data features (see Table 1 below for summary). Figure 1 shows the empirical performance of some of the models in Table 1 using a data for a route in Atlanta, Georgia. The original model of Greenshields clearly doesn't fit the data very well. More complex models with higher number of parameters are a better fit.

Table 1: Speed-Density FD Functional Forms

Table 1 Single-regime Speed-Density Models

Single-regime Models	Function	Parameters
Greenshields Model (1935)	$V = v_f(1 - \frac{k}{k_j})$	v_f, k_j
Greenberg Model (1959)	$V = v_m \log \frac{k_j}{k}$	v_m, k_j
Underwood Model (1961)	$V = v_f \exp(-\frac{k}{k_j})$	v_f, k_j
Northwestern Model (1967)	$V = v_f \exp(-\frac{1}{2}(\frac{k}{k_0})^2)$	v_f, k_0
Drew Model (1968)	$V = v_f[1 - (\frac{k}{k_j})^{n+\frac{1}{2}}]$	v_f, k_j
Pipes-Munjal Model (1967)	$V = v_f(1 - (\frac{k}{k_j})^n)$	v_f, k_j
Newell Model	$V = v_f[1 - \exp\{-\frac{\lambda}{v_f}\{\frac{1}{k} - \frac{1}{k_j}\}\}]$	v_f, λ, k_j
Modified Greenshields Model	$V = v_0 + (v_f - v_0)(1 - \frac{k}{k_j})^\alpha)$	v_0, v_f, k_j
Kerner and Konhäuser (1994)	$V_e = v_f(\frac{1}{1+\exp(\frac{\frac{k}{k_m}-0.25}{0.06})} - 3.72 \times 10^{-6})$	v_f, k_m
Del Castillo Model(1995)	$V = v_f\{1 - \exp[\frac{|C_j|}{v_f}(1 - \frac{k_j}{k})]\}$	v_f, C_j, k_j
Van Aerde Model (1995)	$k = \frac{1}{c_1 + \frac{c_2}{v_f - v} + c_3 v}$	c_1, c_2, c_3, v_f
MacNicholas Model (2008)	$V = v_f\left(\frac{k_j^n - k^n}{k_j^n + mk^n}\right)$	v_f, k_j, n, m

Source: Wang et al. (2013)
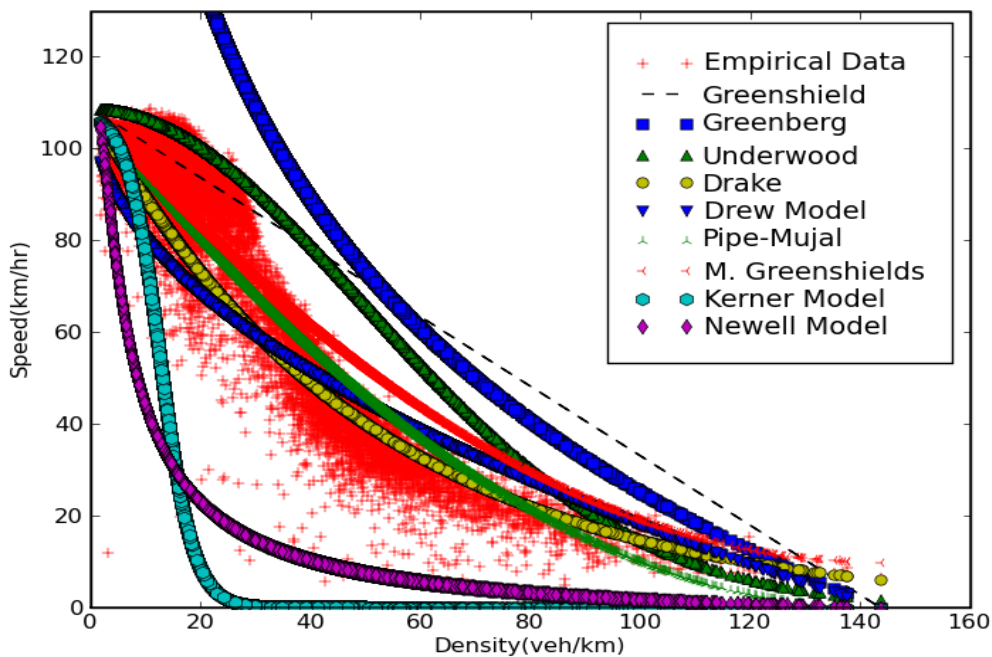
Source: Wang et al. (2013)

Figure 1: Empirical Fit of Single Equation Models for the Fundamental Diagram

The paper by Wang et al. [2013] proposes a flexible specification for the speed-density fundamental diagram. It has major advantages over other existing models:

1. It can fit the data with reasonable accuracy although it is a single regime model.

2. The curve that produces has desirable mathematical properties (e.g. it is smooth and bounded – doesn't go to infinity or zero).

3. The parameters of the model   have a clear interpretation.

The equation for the speed-density relationship is:

$$(1)\ V = v_b + \frac{v_f - v_b}{\left(1 + exp\left(\frac{k - k_t}{\theta_1}\right)\right)^{\theta_2}}$$

We will refer to the above specifications the WLCN model. The parameter v_f is free flow speed and v_b is the average travel speed at stop and go condition. θ_1 is a scale parameter which describes how the curve is stretched out over the whole density range, θ_2 is a parameter which controls the lopsidedness of the curve. The parameter k_t is the turning point that the speed-density curve makes the transition from free-flow to congested flow.

Figure 2 below shows the fit of the model to the same data as in the previous examples.
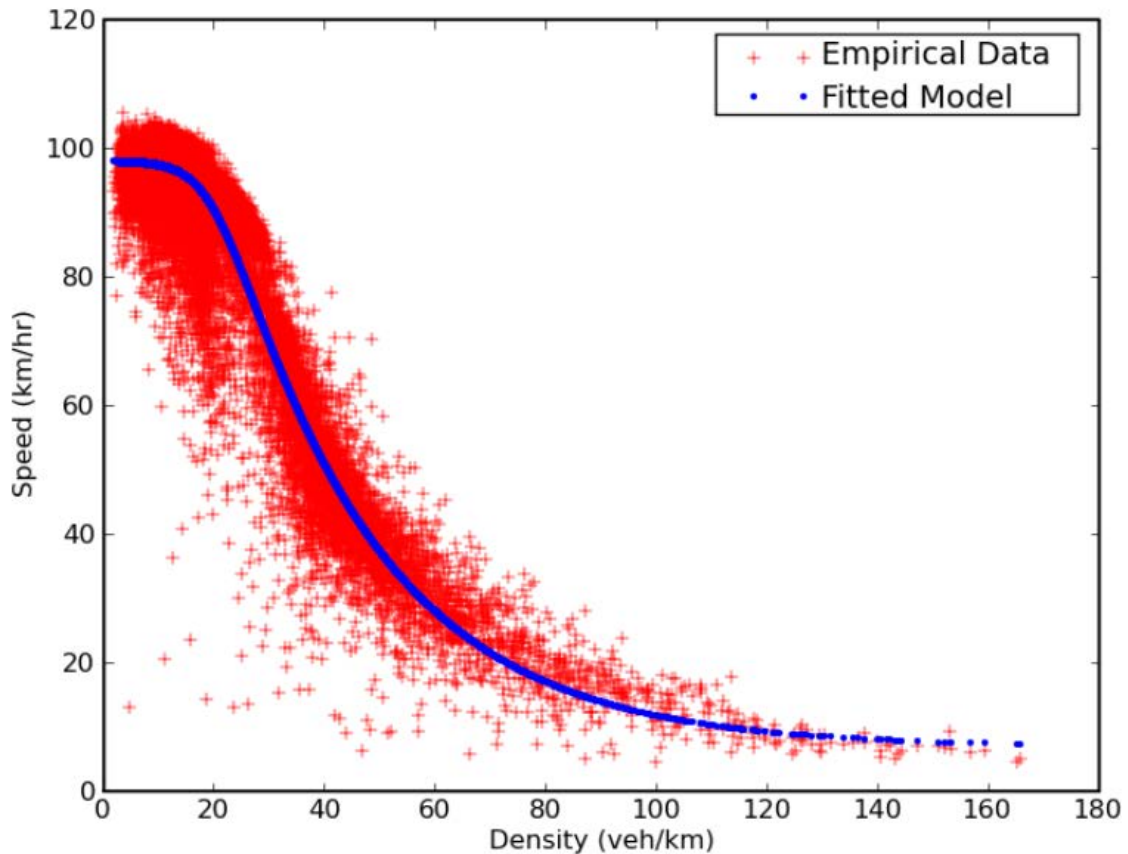
Source: Wang et al. (2013)

Figure 2: Empirical of the Wang et al. (2013) Model for the Fundamental Diagram

Visual inspection of the graph reveals that a very good fit even at very high densities. Authors of the paper test the model over wide range of other traffic data sets, where it is shown to have good a similar performance.

Given the flexibility and interpretability of the model, it was chosen to be used for derivation of the benchmark FD for the new highway. To conclude this section, we use apply the data from a motorway in Brisbane which is expected to have similar traffic conditions as the new motorway. In the next section we describe in detail our methodology for envelope fitting using the sample FD as an example.

From a practical point of view, we choose to work with the flow-density since it was deemed that it is more well-known and accepted type of FD than the speed-density FD. The WLCN model can be easily re-written as a flow-density relationship:

$$(2) \ F = v_b + \frac{v_f - v_b}{\left(1 + exp\left(\frac{k - k_t}{\theta_1}\right)\right)^{\theta_2}} k$$

The equation above is the **mathematical** relationship between the flow and density which describes the **average** empirical relationship between the two variables. We have to infer the parameters of the WLCN model from a set of noisy data.

Usually, this is done by using the Non-Linear Least Squares (NLS) Optimization which can be applied to any type of functional relationship between flow and density:

$$(3) \ F = f(k|\theta)$$

where θ is the set of parameters of the function $f(k|\theta)$. The parameters θ are obtained by minimizing the sum of the squared differences between observed and predicted values for the speed:

$$(4) \ \left(\widehat{F_1} - F_1\right)^2 + \cdots + \left(\widehat{F_n} - F_n\right)^2 \ \text{where} \ \widehat{F_s} = f(k_s|\theta).$$

Efficient algorithms for solving this type of problems such as Levenberg-Marquart method exist (William; Teukolsky 2007)

We estimate the WLCN model for the sample data via the Levenberg-Marquart method implemented in the Matlab optimization toolbox. Note that optimization algorithms usually require an initial value or set of values for the parameters which serve as a starting point in the search for the optimal parameter value.

In the current case, initial values have to be provided for v_f - the free flow speed, v_b - the average travel speed at stop and go condition, k_t the turning point that the speed-density curve makes the transition from free-flow to congested flow as well as for θ_1 and θ_2.

It is important to carefully select the initial parameter values in order to make sure that the global minim of the target function (4) is found instead of a local one. Non-linear optimization is an art and a science at the same time, and it is hard to give a universal recipe which works in all cases.

Visual inspection of the flow-density data in the Figure 3 below strongly suggests that the FD curve will be a simple linear line in all cases. Indeed, if a non-linear fit of an FD curve produces more complex shape this will strongly suggest a mistake in the estimation.

Extensive experiments suggest that the optimal solution of the curve fitting problem is achieved when $\theta_2 = 0$, which means that there is no lopsidedness in the FD curve. In that case, the speed-density FD collapses to a horizontal line with height given by the free flow speed v_f, and the flow-density FD collapses to linear line emanating from (0,0) point with an incline determined by the free-flow speed.

That is, the free-flow speed v_f is the only parameter that can be estimated from the data and its fitted value is $v_f = 74.18 \ km/h$.
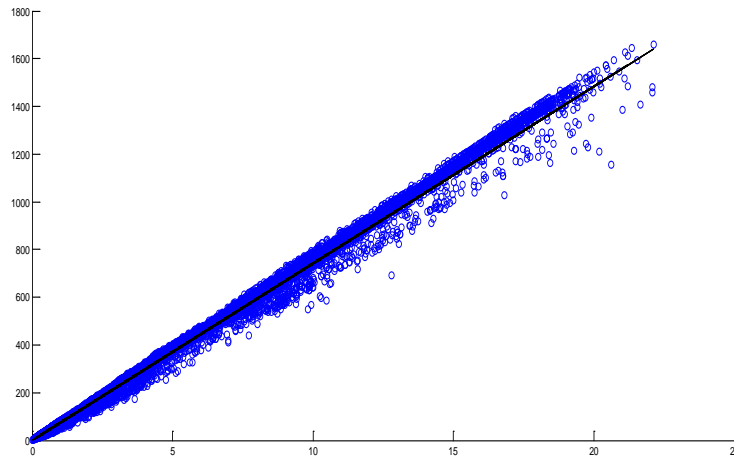
Figure 3: Flow-Density FD for Sample Motorway Estimated from the WLCN Model.

Note that the empirical data doesn't suggest the existence of critical density as the parameter k_t can't be estimated. In this instance, we recommend to use the (expected) critical density figure given by the US Highway Capacity Manual (HCM 2010) of 28 veh/km/lane.

CONSTRUCTING A PERFORMANCE ENVELOPE ON THE FUNDAMENTAL DIAGRAM

In this section we describe in detail how to construct an envelope around the flow-density FD curve. A simple method to extend the envelope when sufficient data points are not available is also proposed. The data and the FD curve estimation from the previous section are used as an example.

We assume that the Fundamental Diagram curve $F = FD(k)$ is known and estimated from the data (se per the previous section). Mathematically, the envelope is defined by two functions $F^+ = FD^+(k)$ and $F^- = FD^-(k)$. The first function defines the upper bound of the envelope and the second function defines the lower bound of the envelope. By definition the following relationship must hold:

$$(5) \quad F^- = FD^-(k) < F = FD(k) < F^+ = FD^+(k)$$

since the FD curve must be positioned between the **lower and upper bands of the envelope.** For a given density level k, the value of $F^+ = FD^+(k)$ determines the upper range of the speed, level which we can assume represents normal traffic conditions. Similarly, and more importantly the value of $F^- = FD^-(k)$ represents the lower range of the speed level which we can assume represents normal traffic conditions. The area encompassed by these two functions represents the normal traffic conditions, area above the envelope represents better than normal traffic conditions, and area below the envelope represents worse than normal traffic conditions.

The KPI trigger area (KPI zone) is the one below the envelope and it is completely determined by the function $FD^- = FD^-(k)$. Obviously, defining the upper bound of the envelope is irrelevant with respect to the derivation of the KPI zone. For completeness and clarity of exposition, we will present both calculations below.

Instead of working with the envelope defined by $\{FD^-(k), FD^+(k)\}$ it would be easier to work with the width of the band. The lower and upper widths are defined by $w^- = FD(k) - FD^-(k)$ and $w^+ = FD^+(k) - FD(k)$, respectively.

The basic idea is to define normal traffic conditions on the Fundamental Diagram as a range of deviations around the Fundamental Diagram curve. This is done using a two-stage process. First, we use a non-parametric approach to obtain an initial envelope around the FD curve. The resulting envelope however is very noisy. To smooth out the noise, a linear function is fitted to the width of the envelope in the first stage, and on that basis the final envelope is computed.

Stage 1: derivation of a non-parametric envelope around the fd curve

Here we describe how to derive the first-stage envelope around the FD. We denote the initial lower and upper envelopes by $FD^{-*}(k)$ and $FD^{+*}(k)$. To fix things, let us consider the splice of the Fundamental diagram representing densities between 0 and 2 veh/km:
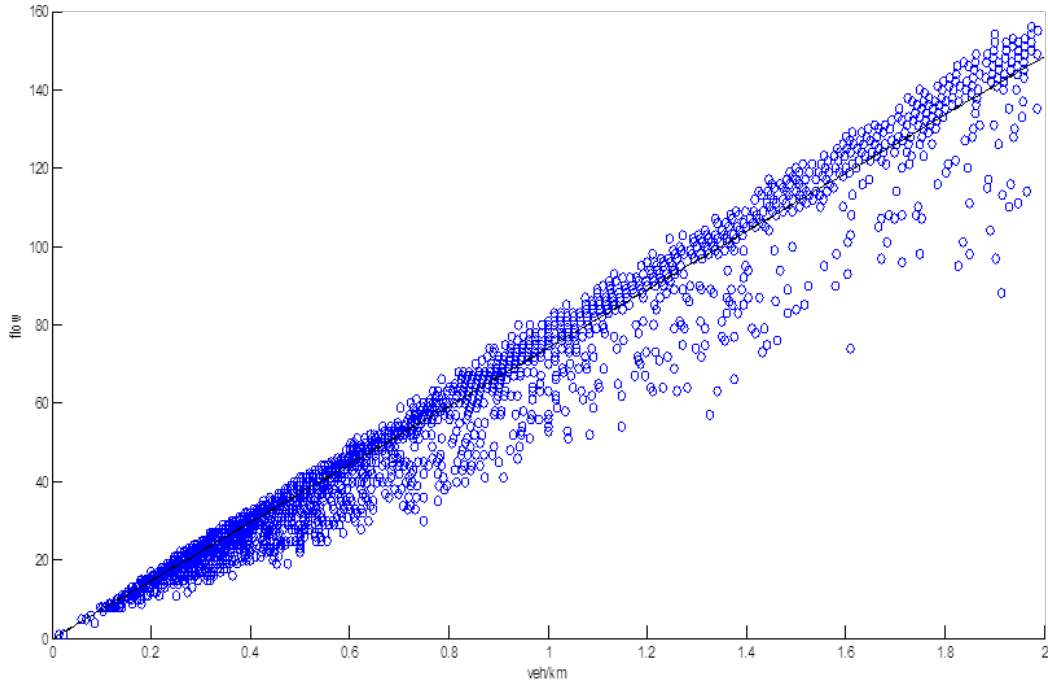

Figure 4: Fundamental Diagram Curve for Densities < 2 veh/km.

Since we are mostly interested in defining the lower bound of the envelope, consider the observations below the FD curve only (later we will briefly comment on defining the upper bound which is done in a similar fashion). There are 1750 observations of flow and density in total: $(d_1, f_1), \ldots, (d_{1750}, f_{1750})$.
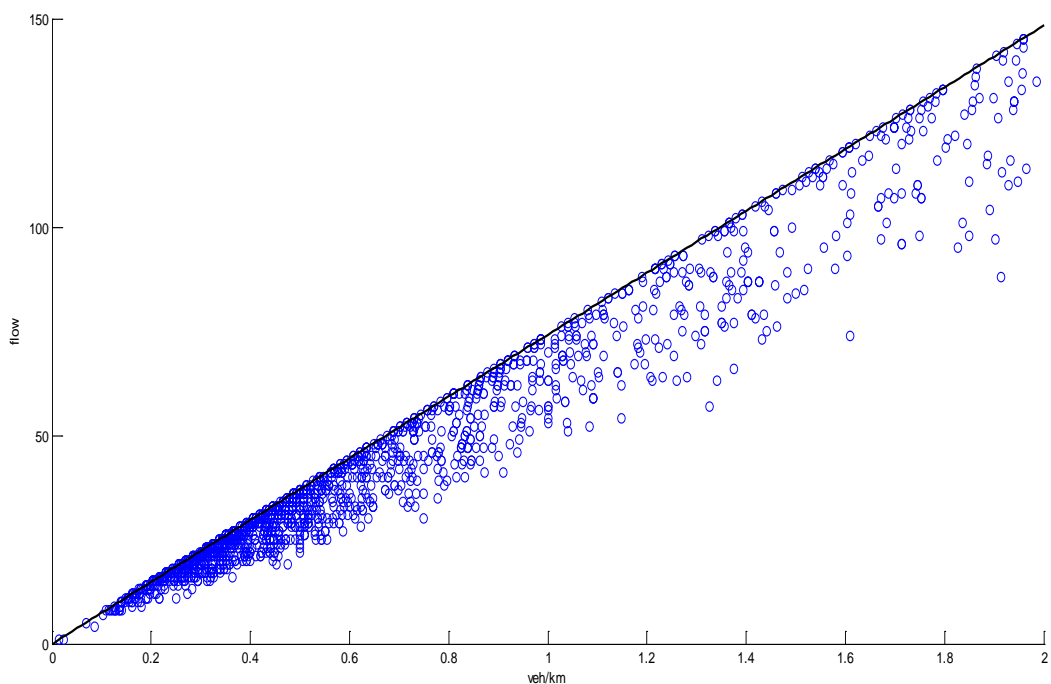

Figure 5: Fundamental Diagram for Densities < 2veh/km and flow rates below the FD curve.

In this particular segment we are interested in defining what is an acceptable level of deviations from the FD curve in terms of distance from it. We start with defining this level for the point in the middle of the segment, which is 1 veh/km.

We consider the set of all distances of the observations below the curve:

$$(6)\ w_1^* = FD(d_1) - f_1, \dots, w_{1750}^* = FD(d_{1750}) - f_{1750}$$

On the basis of these distances for densities in the interval [0,2], we can define what an acceptable distance would be for a flow level when the density is 1 veh/km (middle of the interval). A simple and intuitive way is choose the distance as the upper percentile of the series above. For example, fix the 95% percentile for which 95% of the observations in the sequence (5) (in that case 95% of 1750 is 1662.2 and we take 1663) fall below it.

In the current case, the 95% percentile is 19.62. Thus, the initial lower envelope $FD^{-*}(1)$ at density 1 veh/km is set at:

$$(7)\ FD^{-*}(1) = FD(1) - 19.62 = 74.18 - 19.62 = 54.56$$

Similar approach can be taken to computing the value of the lower envelope for any other density. Mathematically, the method for computing the initial envelope is based on a centred sliding window with length 2. First, we have to fix an end point K on the x-axis where we deem there are sufficient observations to compute the envelope. In the case of our sample data, such point is given by the density value of 22 veh/km. Since the window has length 2, the starting point for the window right corner is 2 and the end point is K. The right corner window is gradually moved from the two end points (2 and K), and the value of the envelope is computed for the mid-point of the window. More specifically, if the right corner of the window is $k \in [2, K]$, then the observations below the Fundamental Diagram curve which fall below the FD curve with densities between $k - 1$ and $k - 1$ are considered:

$$(d_1, f_1), \dots, (d_N, f_N) : k - 2 \le d_i \le k, f_i \le FD(d_i)$$

The 95% percentile value w_k^* of the sequence of distances $w_1^* = FD(d_1) - f_1, \dots, w_N^* = FD(d_N) - f_N$ is computed . Then the value of the lower band of the envelope at the centre of the window is given by:

$$(8)\ FD^{-*}(k) = \max(FD(k) - w_k^*, 0)$$

Note that since the sliding window starts at 2 on the x-axis, then the first point of the envelope which can be computed is for the midpoint of the start which is (2+0)/2=1.

To calculate the envelope in the interval [0,1], the width of the envelope at 1 given by $FD(1) - FD^-(1)$ can be used as the width of the envelope in the beginning:

$$(9)\ FD^-(k) = \max\big(FD(k) - (FD(1) - FD^-(1)), 0\big), k \in [0,1]$$

Similarly, the end point of the sliding window is K and the midpoint of the last sliding window is $\frac{(K+K-2)}{2} = K - 1$. This is the last value at the x-axis, for which the lower bound of the envelope can be computed. To calculate the envelope in the interval $[K - 1, K]$ the width of the envelope at K given by $FD(K - 1) - FD^-(K - 1)$ can be used:

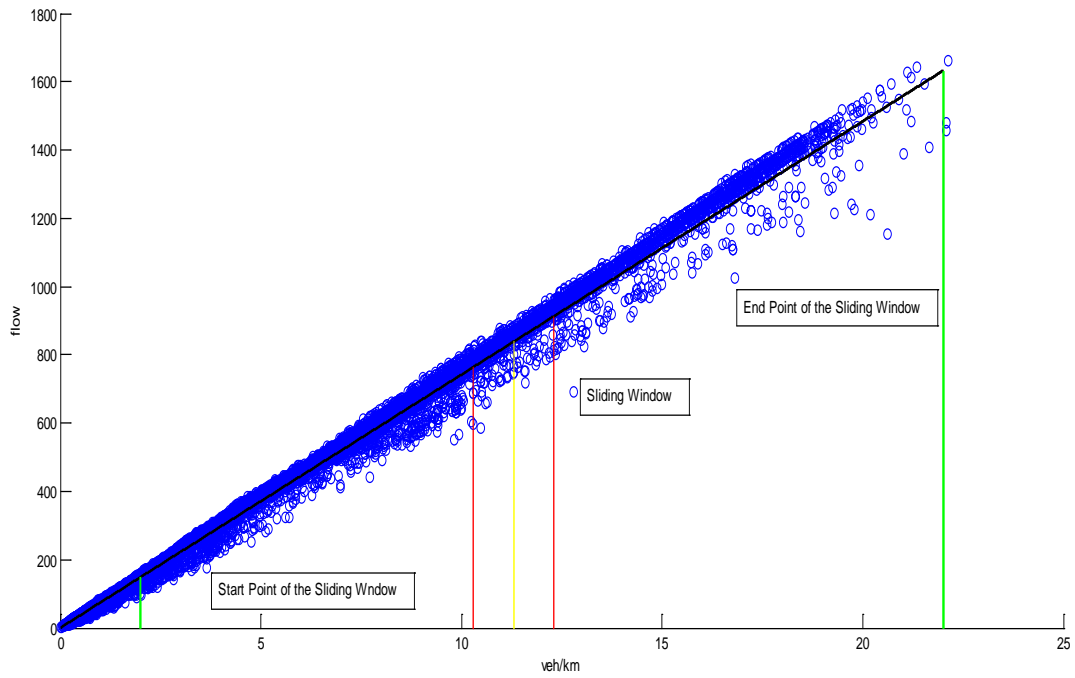$$(10)\ FD^{-*}(k) = \max\big(FD(k) - (FD(K - 1) - FD^-(K - 1)), 0\big), k \in [K - 1, K]$$

Figure 6: Starting Point of the Sliding Window, Sliding Window and End Point of the Sliding Window.

The upper bounds of the envelope can be computed in exactly the same way. For brevity, and given that the exact shape of the upper part of the envelope is irrelevant to the calculation of KPI trigger zones, this is not going to be discussed further. Below is a graphical representation of the result of the procedure above for the initial upper and lower envelopes based on the 95% percentile.
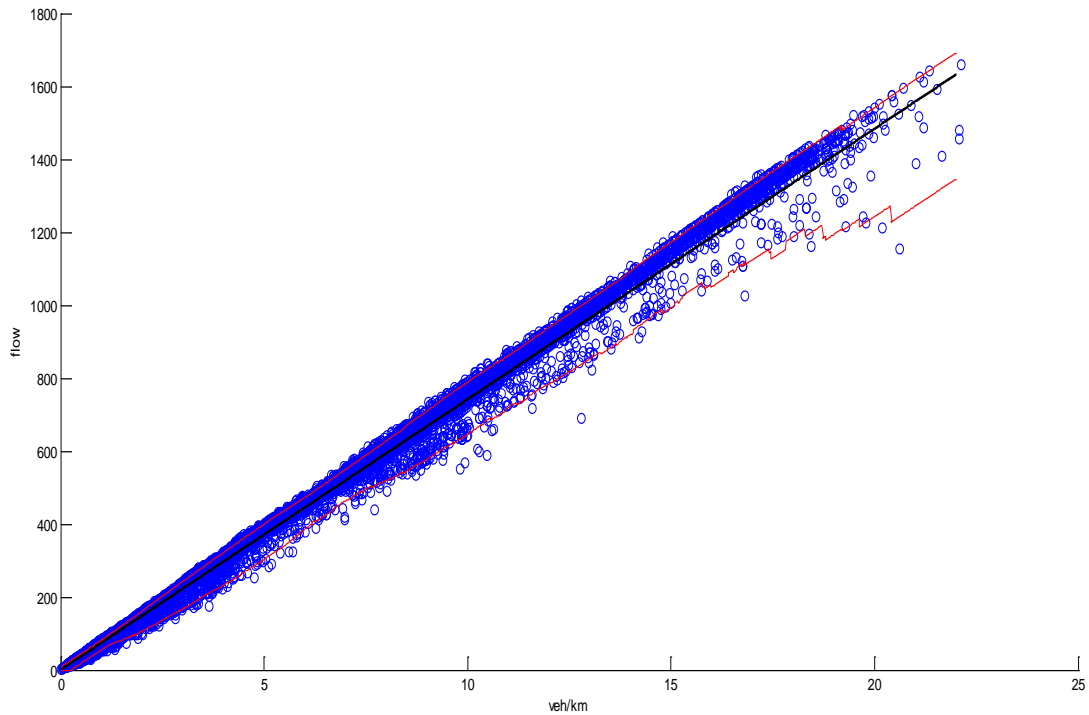

Figure 7: Initial Envelope around the FD Curve based on the 95% percentile.

Stage 2: derivation of a parametric envelope around the fd curve with a linearly expanding width

As it can be seen, the initial envelope is very noisy and tends to show up and down fluctuations around the FD curve. From a practical point of view, we would like to have an envelope with the following two properties:

1. Smooth boundaries which follow the shape of the FD curve
2. Expanding width as the density increases

A natural approach to derive such envelope from the initial envelope is to fit a linear function to the width of the initial envelope and use this function to derive the width of the final envelope. Mathematically, we define the width $width(k)$ of the (lower) envelope as a linear function of the density:

$$(11) \; width(k) = a + b * k$$

The lower bound of the envelope is defined by:

$$(12) \; FD^-(k) = max(FD(k) - width(k), 0)$$

The coefficients a and b are estimated from the initial envelope $FD^{-*}(k)$. The idea is to try to fit a linear function of the type (11) to the widths of the initial lower envelope $FD(k) - FD^{-*}(k)$.

In the current case, we consider the density range of the Fundamental Diagram from 0 to 22 veh/km. We partition this range into small intervals with equal size Δ with endpoints $k_1 < \cdots < k_N$, where $k_i = i * \Delta$. The example here is for interval size equal to 0.01 veh/km which translates into $N = \frac{22}{0.01} = 2200$ intervals (the method is rather insensitive to the number of intervals as long as it is sufficiently large e.g. $N > 100$. The current number of intervals is rather conservative).

We can compute the widths of the initial envelope at the density points k_i: $w_i^* = FD(k_i) - FD^{-*}(k_i)$. A linear regression given by (11) is fit to the widths w_i^* using the standard Least Squares Method. This results in the following estimation for the a and b: $a = 7.56$ and $b = 9.23$. The lower bound of the envelope can now be easily obtained from the formula (12). The same approach is applied to calculating the upper bound of the envelope. The new envelope with linearly expanding width is shown in the graph below:
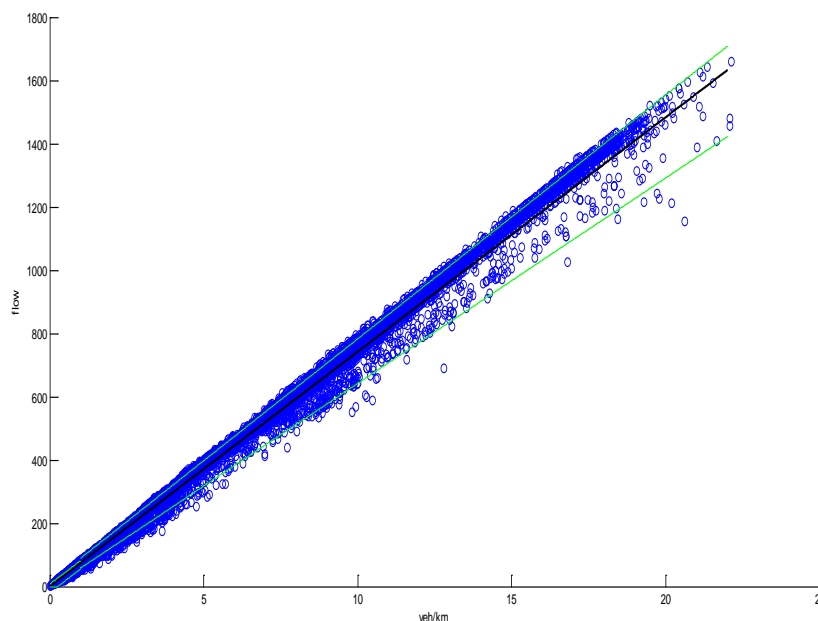

Figure 8: Envelope around the FD Curve based on the 95% percentile.

The envelope bands can be easily extended up to an arbitrary point assuming the linear specification (11) for the width. Let is consider a hypothetical example where the envelope is only defined for densities up to 22 veh/km but the FD curve can be defined (Figure 9) for densities up to 37 veh/km and the curve has a critical density at 28 veh/km.

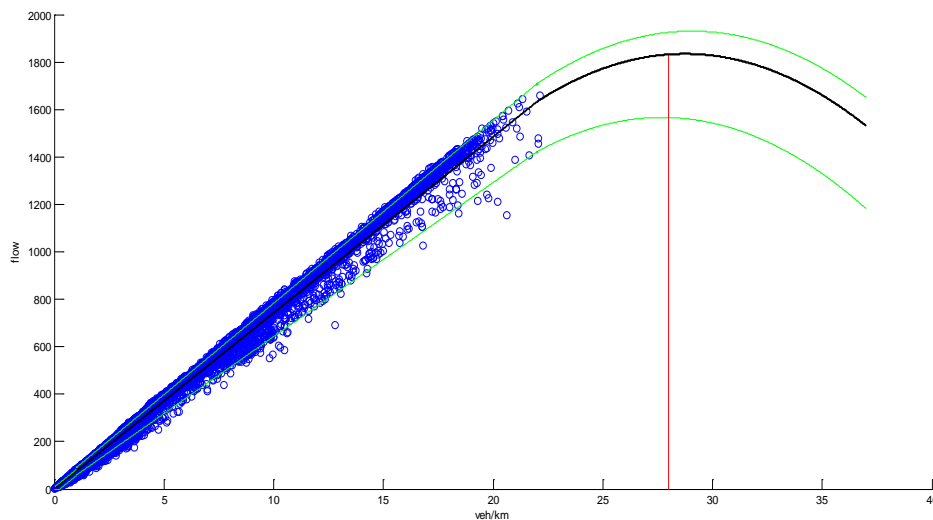Extrapolating the envelope bands using the linear specification results in the envelope shown below in the Figure 9.

Figure 9: Extrapolation of the Envelope around the FD Curve.

CONCLUSION

In this paper we presented a novel approach to using the Fundamental Diagram (FD) for the purpose of calculating travel time KPIs on a new motorway. The methodology is based on the idea that the FD signifies "ideal" equilibrium traffic conditions. Using traffic flow theory and empirical data to define a "performance" envelope around it, we can classify the normal traffic conditions as observations inside the envelope.

Correspondingly, "abnormal" traffic conditions have observations falling outside and below the envelope. In this particular case, the Fundamental Diagram approach was applied as a pre-condition to the underlying travel time requirement for the project, which linked directly to the assumed travel time saving used in the economic business case. By adopting this approach, it removed the risk of the unknown for Leighton Contractors in committing to the contract, while providing the Road Agency with a 'hard' KPI relating to the benefit they sought to achieve through the project.

Our approach is oriented towards implementation for motorway operations. We believe that it can be successfully applied to other existing or new motorways. Besides the novel use of the FD, a useful contribution is the proposed methodology for fitting an envelope around the FD. The envelope construction procedure is technically rigorous and at the same time simple enough that the concept does not require skills in advanced mathematics or statistics to be understood and used.

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