

# STATISTICAL APPROACHES TO INCREASE EFFICIENCY OF LARGE-SCALE MONTE-CARLO SIMULATIONS

David Shteinman,<sup>\*</sup> Thibaud Teil,<sup>†</sup> Scott Dorrington,<sup>‡</sup> Jaslene Lin,<sup>§</sup>

Hanspeter Schaub,<sup>\*\*</sup> John Carrico,<sup>††</sup> and Lisa Policastro<sup>‡‡</sup>

## INTRODUCTION

Many numerical astrodynamics analyses are characterized by a large input space with dispersions on those inputs. They also require numerical integration to propagate orbital trajectories, as well as the spacecraft attitude and actuator states forward in time. Often, Monte Carlo simulations are used, where each sample point is propagated numerically. These features all contribute to long Monte Carlo simulation times. Furthermore, the underlying input-output relationships are nonlinear with many variables interacting with one another. Hence, it is difficult to study the behavior in simulation of output responses as a function of the inputs - as that requires testing of a wide range of input values. Using traditional methods of varying one factor at a time and re-running the whole simulation each time is excessively time consuming. Also, varying one factor at a time means the end user of the simulation's results cannot be certain they have captured the full range of possible input values.

The aim of this paper is to adapt a method for astrodynamics simulations from industrial statistics and empirical modeling, to achieve the following outcomes: **1) Significantly reduce the run time of large-scale Monte Carlo simulations; 2) Ensure the simulation covers a wider range of values/worst case scenarios for significantly less runs than required under standard Monte-Carlo methods; 3) Increase the efficiency of Sensitivity Analysis and Optimization by using a fast/computationally cheap approximate model of the simulation, thus avoiding the need to re-run the simulation to test the effect of alternate input values on the output.**

To achieve outcomes 1-3, we propose to adapt the techniques of *Design & Analysis of Computer Experiments Techniques (DACE)* to astrodynamics simulation and illustrate with two Case Studies.

## DESIGN & ANALYSIS OF COMPUTER EXPERIMENTS TECHNIQUES (DACE)

DACE is an adaptation for simulations of the DOE approach (*Design of Experiments*) used in physical experiments on industrial processes. DOE is a statistical method of experimentation that *varies all inputs in a simulation simultaneously* (rather than one factor at a time) and achieves the following:

- Determines the critical inputs (those with biggest effect on output of interest)

---

<sup>\*</sup> Industrial Sciences Group, Sydney, Australia

<sup>†</sup> Autonomous Vehicles Systems (AVS) Laboratory, University of Colorado, Boulder

<sup>‡</sup> Industrial Sciences Group, Sydney, Australia

<sup>§</sup> Industrial Sciences Group, Sydney, Australia

<sup>\*\*</sup> Autonomous Vehicles Systems (AVS) Laboratory, University of Colorado, Boulder

<sup>†††</sup> Space Exploration Engineering Inc.

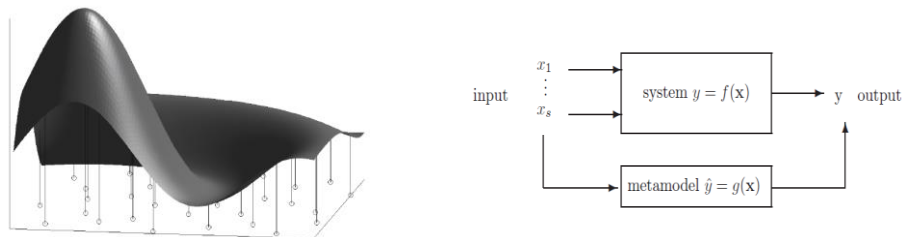
<sup>‡‡</sup> Space Exploration Engineering Inc.

- Quantifies the input/output relationships in an analytical form within the experimental range
- Shows interactions between inputs.

The method of DACE (see Reference 1) has been used extensively in the automotive and other manufacturing sectors, but so far as we know, not in astrodynamics. DACE is used to augment the limited number of runs of a simulation by fitting an approximate statistical model, a surrogate or “Meta-Model”, based on a set of limited observation data acquired by running the simulation at carefully selected design points, generated from a “Space Filling” design. The Meta-model is easier and faster to run than the simulation and is used to predict the simulation performance at unobserved input values. The approximate Meta-model is much simpler than the true one: it approximates the original model as closely as possible and is computationally cheap to compute. The Meta-model is used for sensitivity analysis, optimization and prediction *without needing to re-run the simulation*.

This paper investigates the application and applicability of DACE to two astrodynamics simulations that use Monte Carlo runs: The first is an attitude control simulation using the Basilisk program developed by the *Autonomous Vehicle Simulation (AVS) Laboratory at University of Colorado at Boulder*.<sup>\*</sup> The second is a simulation of trajectory design for the Interstellar Boundary Explorer (IBEX) spacecraft.

In both case studies, we use a five-step process to adapt DACE to astrodynamics simulations (see Reference 2): Step 1) Factor Screening; 2) Design the Simulation “Experiment” using a space-filling design 3) Build a Meta-model; 4) Validating/Checking the Meta-model; 5) Using the Meta-model for Sensitivity Analysis. This process is demonstrated in Figure 1.



**Figure 1. Schematic showing concept of a Space-Filling Design and Meta-model as used in Design & Analysis of Computer Experiments Techniques**

The efficiency gain using DACE is highest when repeatedly running a large Monte-Carlo is a time-consuming process and unfeasible when large number of scenarios are to be tested. Also, DACE is highly effective in cases where it is required to quantify the effect of uncertainties and errors in inputs on some measure of performance.

## SAMPLING METHODS

Monte Carlo (MC) sampling is commonly used to obtain a sample space in a computer simulation. However, as pointed out in Reference 3, while MC sampling is simple to use, a set of MC samples will often leave *large regions of the design space unexplored*. This occurs due to the random and independent nature of the sample sites produced by a random number generator.

<sup>\*</sup> <http://hanspeterschaub/bskMain.html>

To address this problem, in this paper a range of “Space Filling” designs are tested as a way to generate sampling schemes that capture the maximum information between the input-output relationships. The simulation and modeling performance is compared and contrasted using six sampling methods:

1. **Random Sampling** - each input parameter is drawn randomly from the distribution.
2. **Hypercube** - the  $2^n$  vertices of an  $n$ -dimensional hypercube. Hypercube sampling only samples from the vertices of the resulting  $n$ -dimensional hypercube and thus only considers data that are “rare”. Hence it only focuses on the extreme values/events and ignores the rest of the design space. Sampling from the vertices only can be very fast and minimize the experimental runs. However, if the underlying interest lies in the centre of the design space then sampling method such as hypercube will produce samples of limited value.
3. **Latin Hypercube Sampling (LHS)** - has the chief advantage of spreading its runs out over the entire design space. The other main features of LHS are described in detail by References 3-5. In brief: Given an equal number of samples, an LHS estimate of the mean will have less error than the mean value obtained by Monte Carlo sampling. Also, LHS is superior to simply creating a grid over the entire design space as, due to the *Sparsity of Effects principle*, few of the many inputs in a model will prove to be statistically significant. When a computer model exemplifies effect sparsity, variation in the response  $Y$  will be significant only when those few significant inputs are varied. If the significant inputs are kept constant and other inputs varied, the output  $Y$  will vary by a negligible amount (Reference 4, p. 14). LHS is a design that is space-filling in every dimension of the input space and, if collapsed in any one or more dimensions, would not result in duplicate test points.

Other space filling design methods that will be assessed are: methods:

4. **Maximin and minimax statistics**
5. **Uniform designs**
6. **Orthogonal array-based methods.**

Figure 2 shows an example of sample points generated from these methods over a 2D input space.

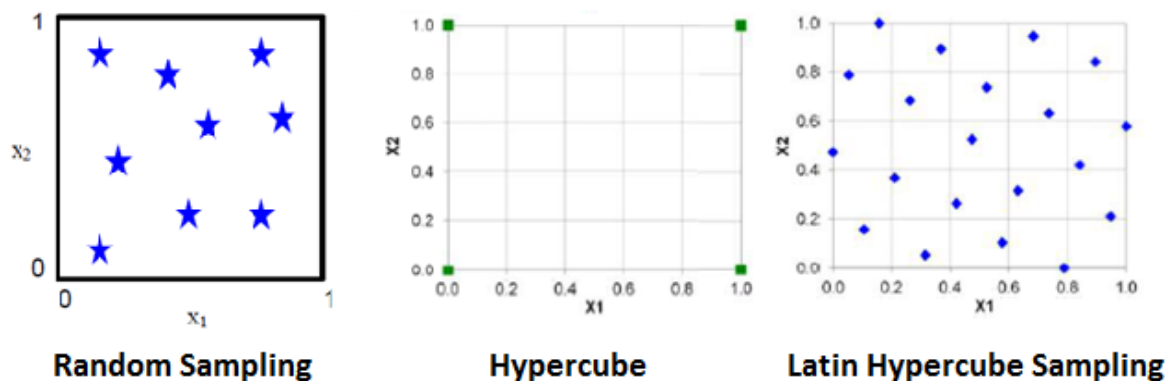


Figure 2. Comparison of Some General Types of Space-Filling Designs in 2-Dimensional Space (References 3 and 4)

## SURROGATE OR “META-MODELS”

The objective is to construct an accurate, simple and easy to run model that approximates the true model of the simulation process.

The initial runs of a sampling design are the “Screening” stage (Reference 6) where, due to the *Principle of Sparsity of Effects*, only a few variables  $X_1, \dots, X_k$  are identified as important for determining the target variable  $Y$  of interest. Each run of the simulation model for specific values  $(X_1^*, \dots, X_k^*)$  will generate a model value  $Y^* = m(X_1^*, \dots, X_k^*)$ , where  $m(\cdot)$  is an unknown function.

The Meta-model is determined by the following two steps:

1. Sampling from a Space Filling Design: Generate a set of input values  $(X_1^1, \dots, X_k^1), \dots, (X_1^n, \dots, X_k^n)$  which is representative of the whole input space and compute the corresponding values  $Y^1 = m(X_1^1, \dots, X_k^1), \dots, Y^n = m(X_1^n, \dots, X_k^n)$ .
2. Model Development: On the basis of the computed pairs of function values and input values  $Y^i, (X_1^i, \dots, X_k^i)$  approximate the unknown function  $Y = m(X_1, \dots, X_k)$ .

For Step 2, three main types of Meta-modeling techniques were compared and contrasted:

1. **Parametric models E.G. Second-order Polynomials:** These models employ a polynomial basis  $x_1^{r_1} \dots x_s^{r_s}$  where  $r_1, \dots, r_s$  are non-negative integers. The number of polynomial basis functions dramatically increases with the number of input variables and the degree of polynomial. (Reference 1, p.27). For our Meta-modeling purposes – we note an issue with Polynomials is that for modeling large and complex simulations there may be many local minimums/maximums within the design space. This requires a high-degree polynomial to approximate the true model (Reference 1, p.27). In such cases collinearity between inputs becomes a serious problem.
2. **Radial Basis Functions:** Radial basis function (RBF) methods are techniques for exact interpolation of data in multi-dimensional space (Reference 7 and Reference 1, p.177-179). The RBF maps the inputs to the outputs using a linear combination of the basis functions. We tested an RBF interpolation with cubic and thin plate basis functions.
3. **Non-parametric methods E.G Random Forests** – have no formal distributional assumptions (hence a “non-parametric” regression), and automatically does screening/variable selection. This is very useful in astrodynamics simulations where there a large number of input variables and need to reduce the dimensionality to the critical inputs with most significant effect on the output response. Random forests can also handle highly non-linear interactions.

The alternative modeling methods are assessed in terms of making satisfactory predictions at untried points. The chosen Meta-model is verified by using the *Root Mean Square Error* (RMSE) of prediction at untried points. The smaller the RMSE value the better the Meta-model. The RMSE (square root of the MSE) represents the departure of the Meta-model from the true model.

## CASE STUDY 1: ATTITUDE CONTROL SIMULATION

The first case study applies the methods of DACE to an attitude control simulation of a tumbling spacecraft using the Basilisk software package developed by the *Autonomous Vehicle Simulation (AVS) Laboratory at University of Colorado at Boulder*. For a given initial state (attitude and rotation rate) and spacecraft

design parameters, the software simulates a sun-pointing feedback control problem to determine the motor torques controls of the three orthogonal Reaction Wheels (RWs) to achieve a final target state, with zero rotation rate.

The goal of the simulation is to assure that, given a range of initial states and uncertainties of real-life missions, a specific spacecraft design can achieve the de-tumbling maneuver while fulfilling a set of mission requirements. For this scenario the following mission requirements are assumed for an acceptable maneuver:

- The attitude will enter and remain within 1° of the goal frame within 5 minutes
- The RW speeds will not exceed 3000 RPM during the maneuver.

### Inputs & Dispersions

The model contains 29 inputs, describing the initial state of the spacecraft, the initial rotation rates of the reaction wheels, and a set of spacecraft parameters. Each input has dispersions with either uniform or normal distributions (detailed in Table 1).

**Table 1. Dispersions on the 29 inputs.**

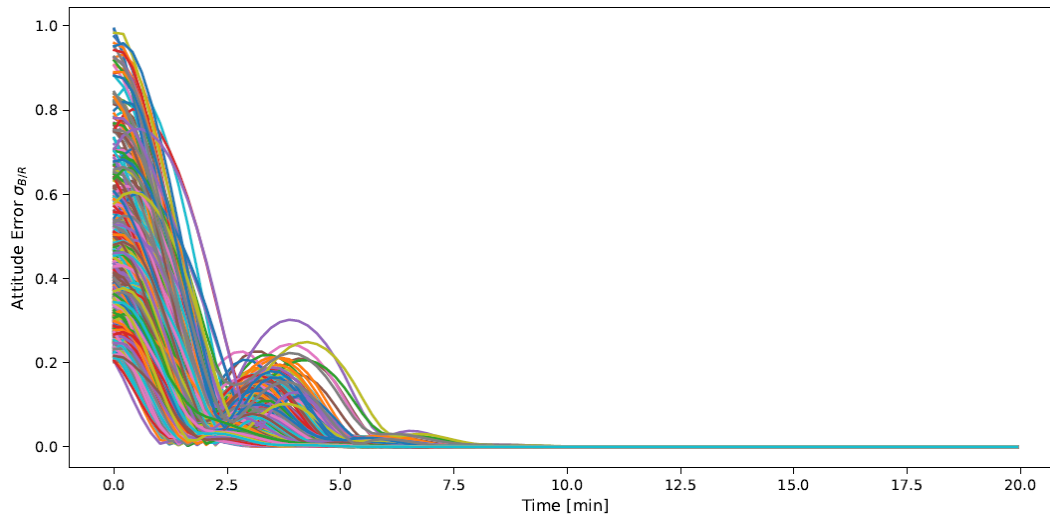
Input Factor	Description	Distribution	Parameters	Units
Inertial Attitude (x,y,z)	Initial attitude of spacecraft (Euler angles with respect to inertial frame).	UNIFORM	[0,2 $\pi$ ] (on each component)	rad
Inertial Rotation rate (x,y,z)	Initial rotation vector in inertial frame.	NORMAL	Mean 0, std dev 0.25 deg/s (on each)	deg/s
Mass	Mass of the Spacecraft	UNIFORM	Bounds [712:5; 787:5], mean 750	kg
Centre of Mass Offset (x,y,z)	Position of Centre of mass	NORMAL	Mean [0,0,1], std dev [0.0017,0.0017,0.0033]	m
Inertia Tensor	3x3 diagonal inertia tensor	NORMAL	Mean diag(900,800,600), std dev diag(0.1,0.1,0.1)	kg.m <sup>2</sup>
Inertia Tensor Rotation Angle (x,y,z)	Rotation of inertia tensor by 3 Euler angles	NORMAL	Mean 0, std dev 0.1 deg (on each)	deg
RW1 axes	Spin axis of reaction wheel 1	NORMAL	Mean [1,0,0], std dev [0.0033,0.0017,0.0017]	-
RW2 axes	Spin axis of reaction wheel 2	NORMAL	Mean [0,1,0], std dev [0.0017,0.0033,0.0017]	-
RW3 axes	Spin axis of reaction wheel 3	NORMAL	Mean [0,0,1], std dev [0.0017,0.0017,0.0033]	-
RW speeds	Initial rotation speed of the 3 RWs	UNIFORM	RW1: [95,105], RW2: [190,210], RW3: [285,315]	RPM
Voltage to Torque Gain	Gain in commanded voltage to actual torque of RWs	UNIFORM	Bounds: [0.019,0.021], Mean 0.020	-

## Outputs

The performance of the de-tumbling maneuver is determined by five output parameters, measuring the rotation rates of the three RWs, and the times taken for the attitude and rotation rate to settle to their targeted states.

## Sampling

To cover the input space, 1000 sample points are produced using the LHS sampling method with dispersions on the 29 parameters. For each sample point, time-series data is produced for the five output parameters over the duration of the de-tumbling maneuver, from which the maximum values were determined. Figure 3 shows a time-series plot of the attitude error (from the targeted attitude) for all 1000 sample points using the LHS.



**Figure 3. Time-series data of the spacecraft attitude for the 1000 sample points.**

## Meta-Model

The inputs and outputs from these 1000 points are used to produce a Meta-model that is able to predict the outputs of any untested point in the input space. Several types of models are tested: non-parametric (Radial Basis Functions and Random Forests) and parametric models (E. G Linear Regression, Quadratic Polynomials).

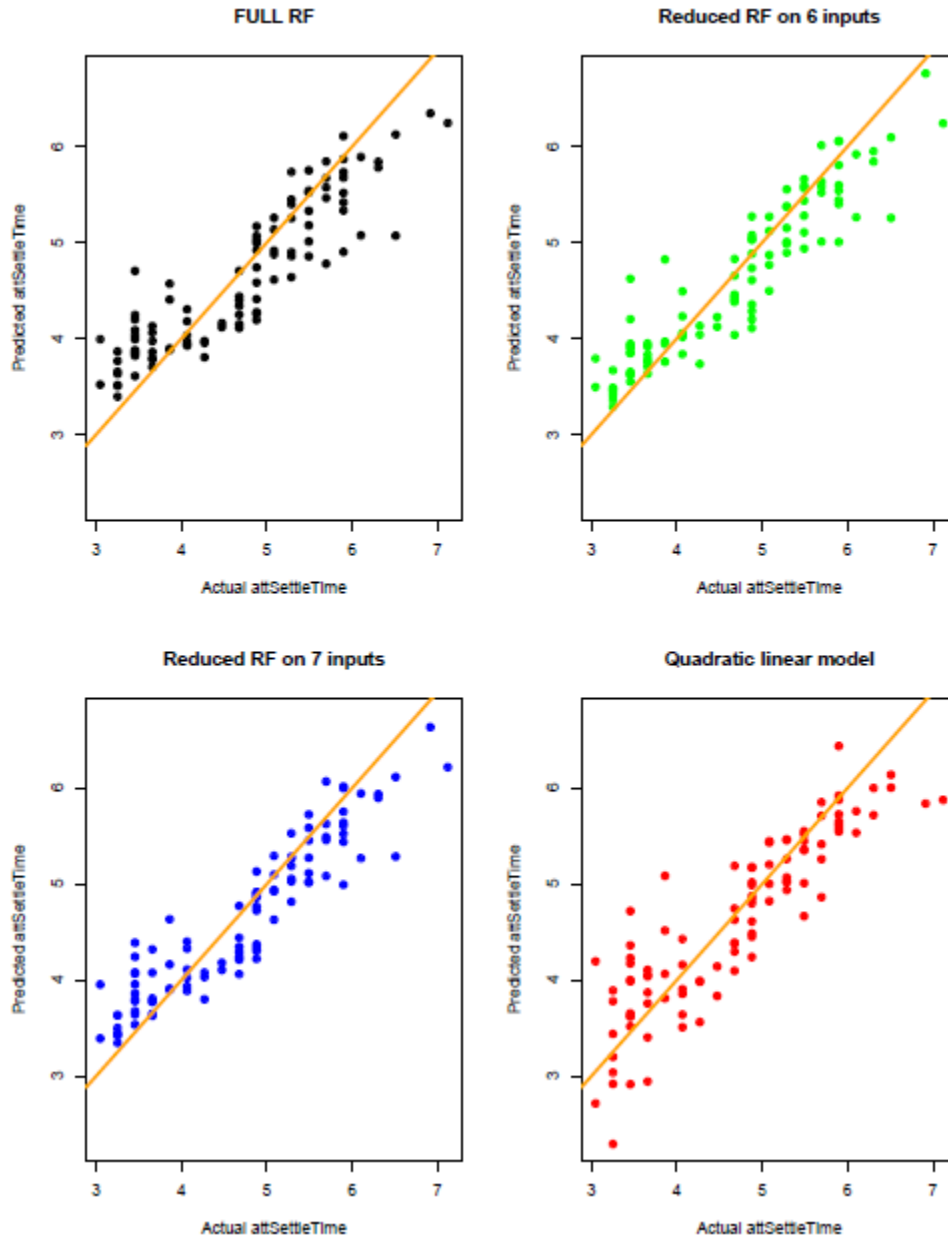
Random Forest (RF) is an effective machine learning model for predictive analytics. It is chosen as the preferred model due to its ability to simultaneously perform automatic variable selection on the large number of inputs (29) in the attitude control simulation. As RF is a non-parametric method it does not assume any distributions, making it ideal for a large and complex simulations (as in astrodynamics) characterized by a large input space, and an unknown input/output relationship that is expected to be highly non-linear.

The first RF model is the full model, where the regressing the output variable against all 29 input variables and draw a variable importance plot and calculate predicted values for the observations in the testing set and get RMSE as the discrepancy measure. The second model is fitted using the top 6 variables only, as shown in the variable importance plot and then repeat the above step. The third model is fitted using the top 6 variables plus 1 variable that is not in the top 6 but is considered to

be of practical importance in predicting the output variable (combined with expert's opinion) and then repeat the above step.

To test the predictive power of each method, a Meta-model was produced from 900 of the sample points. This Meta-model was used to predict the outputs of the remaining 100 sample points and compared to the actual results generated from the simulation. Figure 4 shows a plot of the actual vs the predicted *attitude settle time* for the 100 points for the three Random Forest models, plus the Quadratic Polynomial. From the plots, it can be seen that there is a spread in the predicted attitude settle time. The goodness of the fit was measured by the *root mean square error* of the residuals.

Table 2 below shows the RMSE for 4 Meta-models. The "best" model (lowest RMSE) was the Random Forest model of 6 inputs.



**Figure 4. Predicted vs. Actual outputs for various Meta-Models**

**Table 2. RMSE for various Meta-Models**

RMSE	
RF full model	0.357
RF reduced 6 inputs model	0.3178
RF reduced 7 inputs model	0.3186
Quadratic linear model based on 7 inputs	0.4685

### Significant Inputs

Due to the *Principle of Sparsity of Effects*, it was expected that only a few inputs will be statistically significant. This was the case as Figure 5 shows the strength of the significance of each of the inputs, expressed as a function of the branching of each input in the Random Forest “Tree”. From the results, it can be seen that the *initial state of the spacecraft (initial attitude and rotation rate)* are the most significant inputs. This makes sense, as these values determine the change in attitude and rotation rate that the RWs need to remove to achieve the final target state. The spacecraft mass and inertia parameters are expected to have small variances, as any large variances would be easily identified during spacecraft assembly and testing.

To test the effects of the other 23 remaining inputs, a new set of inputs is generated with a fixed initial attitude of  $[2\pi/3, 2\pi/3, 2\pi/3]$  and a fixed initial rotation rate of  $[0.2, 0.1, -0.1]$  deg. (This initial state was chosen to give a difficult state to recover from.) A new set of 1000 sample points is generated from a LHS with dispersions on the remaining 23 parameters. The new sample points show little variation of the 5 output parameters, confirming that, as expected, the initial attitude and rotation state have *the most significant effect on the outputs*.

Following the initial attitude and rotation rates, the next significant inputs are found to be the *initial rotation rates of the three reaction wheels, followed by the Voltage to Torque gain*.

The same plot (see Figure 6 below) with initial states removed, shows a different ordering of the other parameters (Torque gain is higher). This suggests that the ordering of the significance of the other parameters is dependent on the initial state of the spacecraft.

The effects of the remaining 23 parameters are still much smaller than the initial states. These parameters describe properties of the spacecraft such as the location of the centre of mass, and the balancing of the spacecraft relative to its body frame. The variances on these parameters are small, as they are determined by the tolerances in construction.



Full model attSettleTime..s.

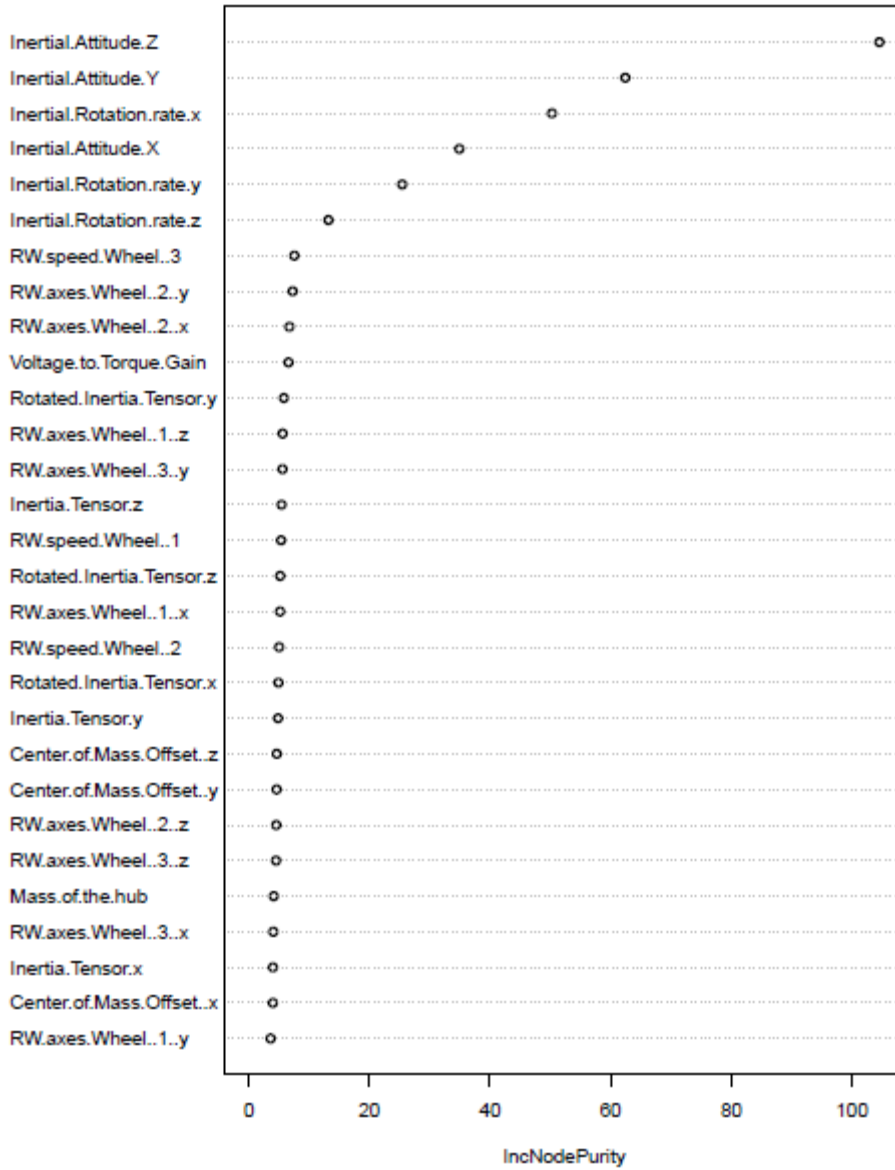
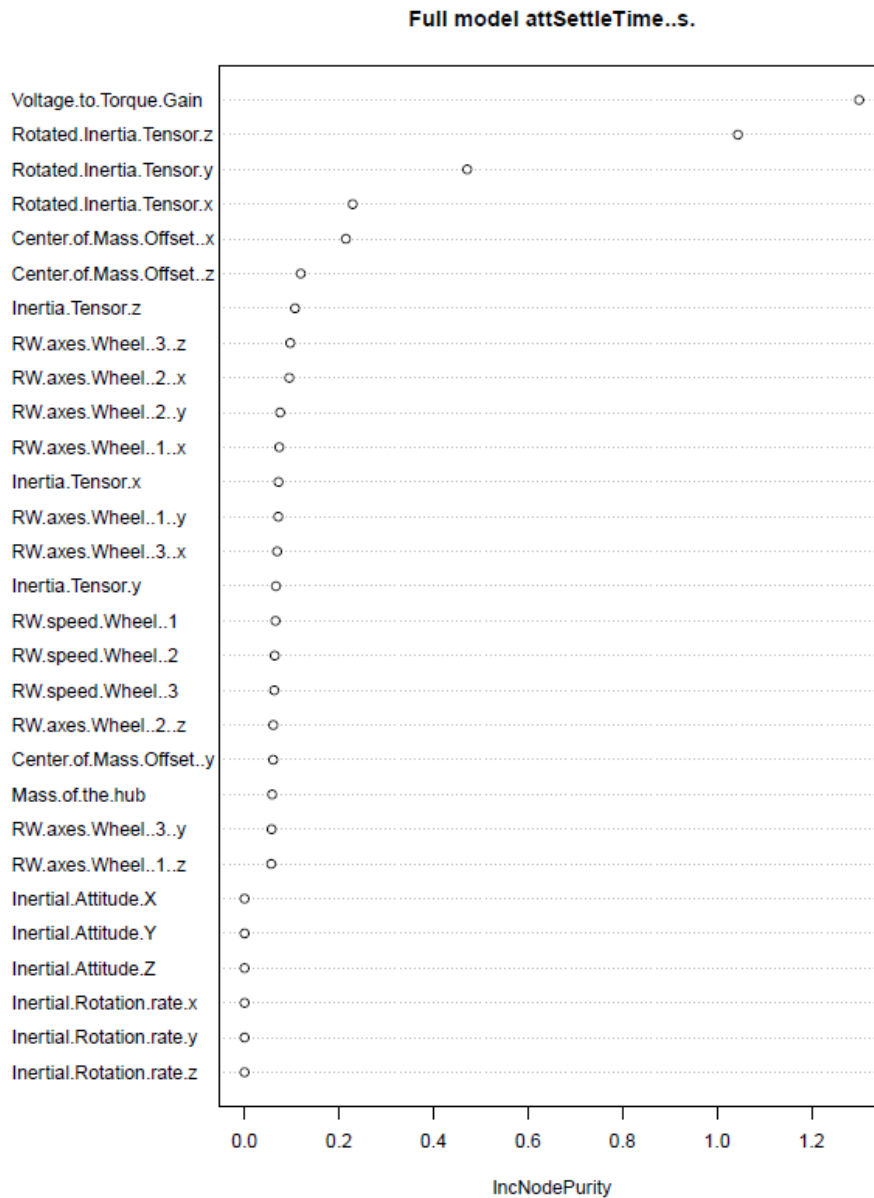


Figure 5. Variable selection plot showing the significance of each input



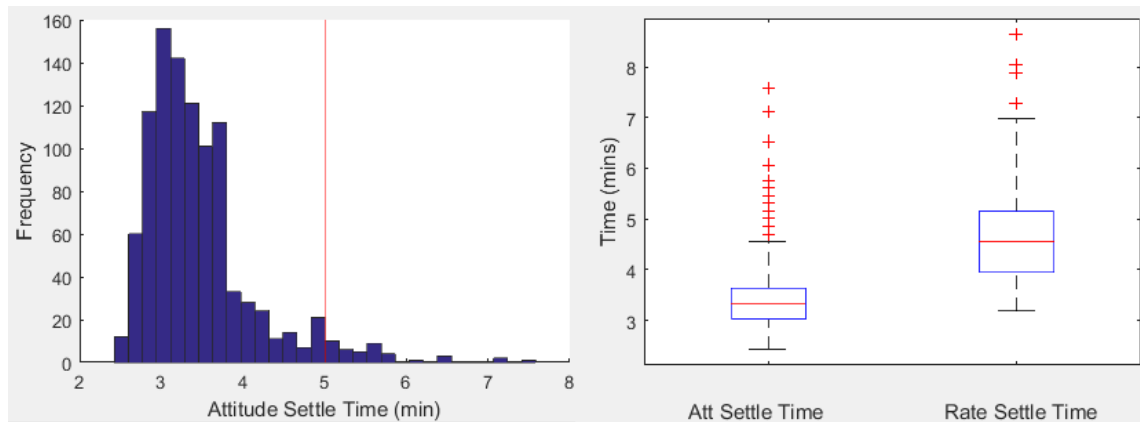
**Figure 6. Variable selection plot with the initial states removed**

### Exploratory Data Analysis

Exploratory Data Analysis was performed on the results produced from the 1000 sample inputs to identify input values that exceeded the mission requirements. Figure 7 shows a histogram and box and whiskers plot of the attitude and rate settle times. The results show that 95% of the sample points had attitude settle times within the 5-minute constraint.

These results suggest that the maneuver would not meet a 3-sigma success rate per the requirements. During mission design and testing, this analysis provides critical insight on the bounding cases and on the sensitivity to input parameters. This leads to a better understanding of the space-

craft's dynamics and capabilities. In this specific scenario, the simulation would first be validated, and the inputs would be examined to ensure realistic values for this maneuver. If the success rate remains low, the design of the maneuver or the spacecraft could be modified. The requirements could also be put into question as slightly relaxing the required attitude settle time would yield positive results.



**Figure 7. Histogram and Box and Whiskers plot of attitude settle time for the 1000 points**

Only a single input is found that exceeded the 3000 RPM limit of the RW speeds. The inputs for this point are found to have a large 10-sigma draw on the z component of the centre of mass location (an error of -35.9 mm from the mean). This error would likely be picked up during inspection of the spacecraft prior to launch, so is unlikely to be encountered during a mission.

In order to further explore the input space, a new set of 10,000 sample points is produced, using the same sampling method. This produced two additional outliers that exceeded the maximum RW speed requirement. The inputs for these points showed no significantly large draws on any of the inputs (the largest was a 2-sigma draw on the y component of the initial rotation rate).

It is likely that the unique combination of input values for these two points lead to the large RW speed outputs. In future analysis, the Meta-model will be used to explore the region surrounding these points to find the extent on the region in the input space that leads to outputs that exceed the mission constraints.

### Sensitivity Analysis

The analysis of outliers described above are limited to the output values produced from running the finite sample inputs. While the increased 10,000-point set produced additional outliers, the computation time increased significantly (by approximately 8 hours). Running additional sample points to further explore regions of interest would lead to long run times. We may use the Meta-model produced to rapidly explore the input space, without needing to re-run the simulation.

Using the RF Meta-model, a Sensitivity Analysis is performed to test the range of outputs *within a wider input space* to see if the mission requirements are still met. Sets of points are generated to test extreme input cases that may be encountered during a mission. Two sets of extreme points are tested in the initial analysis:

A. Initial speeds of 500 RPM on all 3 RWs, as well as a +3-sigma variations on the remaining parameters.

B. A Hypercube with  $\pm 3$ -sigma variation on the initial rotation rate, and +3-sigma variation on the remaining inputs (a total of  $2^3 = 8$  points)

The attitude settle time of set A was found to be 6.166 minutes, exceeding the 5-minute threshold. This value was found to be in the upper range of the results from the 1000 tested, with only 3 points exceeded this value. Despite the increased initial speeds of the three RWs, the maximum speeds were all found to be below the 3000 RPM threshold (the maximum was found to be 1628 RPM on RW2).

The results from set B showed that none of points exceeded the mission requirements on attitude settle time or RW speeds. This suggests that input regions that exceed the mission requirement are not determined by maximum values on each of the input parameters but are instead determined by specific combinations of values that may be non-deterministic. This confirms our expectation that *the Hypercube sampling method may not be sufficient* to ensure that none of the inputs exceed the mission requirements.

### **Future work**

Further testing of the RF Meta-model will test input values expected to lead to extreme output results that may exceed mission requirements. This Sensitivity Analysis will be used as a low cost /rapid development of the operating window of the entire system under a wide range of worst case scenarios.

In particular, the Meta-model will be used to search regions in the input space that exceed the constraints. By analyzing the outliers in the data – we may discover regions around the outlier points, where all points within that region will exceed the bounds.

### **CASE STUDY 2: MISSION ANALYSIS FOR IBEX**

The Interstellar Boundary Explorer (IBEX) mission is a NASA mission that was launched in 2008 with a goal of mapping the boundary of the solar system (Reference 8). At the conclusion of prime mission, IBEX performed a series of maneuvers to change its orbit for extended mission. Prior to launch, the flight dynamics team for the IBEX mission performed a covariance analysis on a number of mission segments including the pre-launch trajectory design, post-launch orbit verification, post-launch ascension planning, and long-term orbit evolution.

The purpose of the analysis was to ensure that given uncertainties in the initial state vector and uncertainties in the magnitudes of each maneuvers, the resulting trajectories would meet a set of mission requirements. The main mission objectives were that the orbit's radius of perigee does not drop below 2.3 Earth radii, and that the eclipse time in Earth's shadow does not exceed 4 hours.

For each of these mission scenarios, a set of perturbed initial states were generated using two sampling methods: First, a traditional Monte Carlo simulation with 1000 random draws from the 3-sigma covariance ellipse, and secondly, the 3-sigma vertices of a Hypercube with 128 points. Each sample point was propagated using AGI's Systems Tool Kit (STK), producing time-series data of the perigee radius, eclipse time, and other trajectory parameters. This data was used to confirm the mission design would meet the trajectory constraints for a specified variance on the input parameters. The details of this analysis were presented in Reference 9.

### **Mission Scenario**

In this case study, we repeat the same analysis as that presented in Reference 9, focusing on the final extended mission of IBEX (Reference 10). Following its primary mission, the spacecraft performed a series final orbit raising maneuvers to place it into a highly elliptical orbit of period 9.1 days.

The inputs to the simulation are the initial state of the spacecraft prior to the final orbit raising maneuver, and the magnitude of the delta-V of the final maneuver. Perturbations are added to each of these 7 inputs to account for the uncertainty of the initial state, and the performance of the maneuver execution. The uncertainty in the initial state is described by a 6x6 covariance matrix, generated from orbit determination. The coordinates of the perturbed states are expressed in the frame of the principle axis of this covariance matrix, found through a Cholesky decomposition.

The uncertainty in the delta-V of the maneuver is normally distributed with mean a mean around 77 m/s with an uncertainty of about 3%. (The uncertainty is due to the uncertainty in the predicted tank pressure at the time of maneuver ignition.)

### **Application of Sampling Methods**

The previous analyses (Reference 9) generated two sets of perturbed initial states using 1) Random sampling over 1000 points and 2) Hypercube sampling on a 7-dimensional Hypercube with vertices at  $\pm 3$ -sigma in each input.

Similar to Case Study #1 above, in this Case Study we will test a range of space-filling designs, including:

- Latin Hypercube Sampling (LHS)
- Maximin and minimax statistics
- Uniform designs
- Orthogonal array-based methods

Each of the sample points is propagated forward in STK for 1014 orbits (approximately 25 years) producing time-series data of the perigee and eclipse times. The objective is to confirm that for any perturbation of the initial state, the trajectory will meet constraints on the minimum periapsis radius and maximum eclipse time.

### **Expected Results**

The Hypercube contains points with combinations of 3-sigma draws on each of the input parameters. It is expected that these points represent the extreme cases of the inputs that are likely to occur in the mission, and thus lead to extreme output values. However, the Hypercube method only covers the boundaries of the input space. We expect that there may be *internal regions in which combinations of the inputs generate outputs that exceed the mission bounds* (as was found for the RW speed outputs in Case Study 1).

In the previous analysis, the Hypercube sampling method was chosen for its simplicity, and the 3-sigma value was chosen arbitrarily. We will suggest alternative space filling methods which will give greater confidence that we will more efficiently sampling the entire input space, as the Hypercube sampling does not necessarily cover the entire input space. While the outputs of all these runs all met the mission constraints, there is no guarantee that there is not any "convexity" in the input space - e.g. regions where the outputs do exceed the constraints.

Using the LHS and other space-filling methods, we expect to cover the input space in a wider range than the Hypercube, such that we may be more confident that there are no convex regions that exceed the bounds. We expect this can be achieved with fewer sample points than simple random sampling method, with a significantly lower computation time.

The initial study of IBEX was produced early in the IBEX mission. The IBEX mission is now in its 10th year, with ephemeris data available. The final analysis we will perform will be to compare

the actual time-series results to those predicted by the model. This will confirm the accuracy of the propagation.

## REFERENCES

1. T. Fang; R. Li; A. Sudjianto, *Design and Modelling for Computer Experiments* Chapman & Hall, 2006
2. D. Shteinman; Z. Lazarov, Y. Ting, Y (2017) "Use of Advanced Statistical Techniques for Mission Analysis: Case Study from A Google Lunar X Team (SpaceIL) " *AAS/AIAA Astrodynamics Specialist Conference*, August 2017, AAS/AIAA 17-664
3. A. Giunta; S. Wojtkiewicz; M. Eldred (2003) "Overview of Modern Design of Experiments Methods for Computational Simulations" *AIAA 2003-0649, 41st AIAA Aerospace Sciences Meeting*
4. J. Glanovsky (2013) "An Introduction To Space-Filling Designs" *ASQ Statistics Division Newsletter*, Vol.32, No. 3, pp.12-18
5. M.D. McKay; R.J. Bekman, W.J. Conover (2000) "A Comparison of Three Methods for Selecting Values of Input Variables in the Analysis of Output From a Computer Code", *Technometrics*, Vol.42:1, 55-61
6. B.Jones, C. Nachtsheim (2011) "A Class of Three-Level Designs for Definitive Screening in the Presence of Second-Order Effects", *Journal of Quality Technology*, Vol.43, No. 1, January 2011, pp. 1-15
7. P. Chandrashekarappa; R. DuVigneau (2007) " Radial Basis Functions and Kriging Metamodels for Aerodynamic Optimization", *INRIA Research Report 00137602v1*
8. McComas, et al., (2005) "The Interstellar Boundary Explorer (IBEX) mission"
9. J. Carrico, L. PolICASTRI, R. Lebois & M. Loucks (2010) "Covariance Analysis and Operational Results for the Interstellar Boundary Explorer (IBEX)", *AIAA Astrodynamics Specialist Conference*
10. J. Carrico Jr., D. Dichmann, L.PolicASTRI, et.al. "Lunar-Resonant Trajectory Design For The Interstellar Boundary Explorer (IBEX) Extended Mission," Paper No. AAS 11-454, *AAS/AIAA Astrodynamics Specialist Conference*, Girdwood, Alaska, USA, August 2011