

# Introducing ISG

## Use of advanced Kalman filtering and statistical techniques for error correction and positioning accuracy in Geoscience Australia's Ginan software toolkit

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Simon McClusky (Geoscience Australia)

*May 2022, Locate Conference, Canberra*

**ISG** The Industrial  
Sciences Group

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North Sydney, NSW 2060

Australia

# Industrial Sciences Group

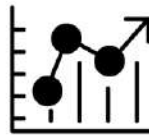
15 years' experience in implementing state-of-the-art research in advanced analytics to deliver commercial outcomes

## 8 Core Capabilities all under 'One Roof'

**Multi-disciplinary** and  
**multi-sector** approach

with skills and expertise in:

Advanced Analytics



Astrodynamics



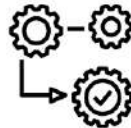
Space Situational  
Awareness



GNSS/Geodesy



Process Optimisation



Simulation



Machine Learning

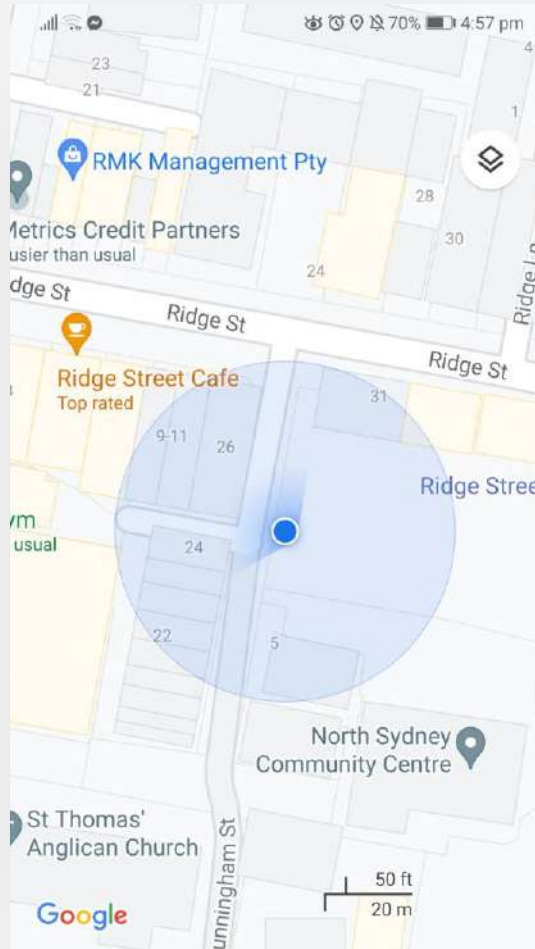


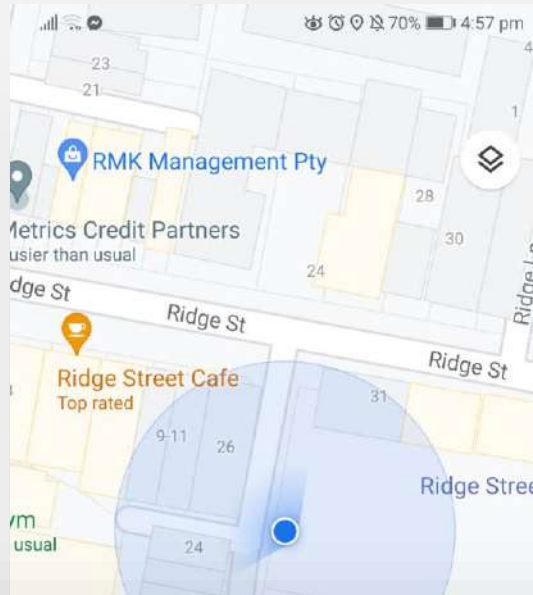
Scientific Programming



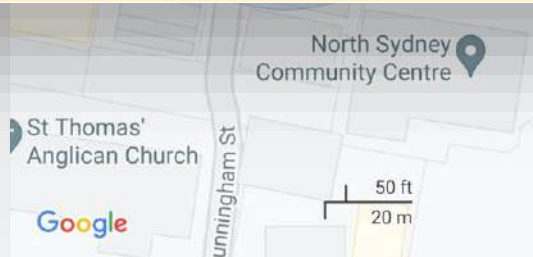
# Outline

<b><i>Section 1</i></b>	<b>The Ginan software toolkit ISG's contributions to Ginan</b>
<b><i>Section 2</i></b>	<b>Smoothing</b>
<b><i>Section 3</i></b>	<b>First-order Gauss Markov modelling</b>





**“How do I improve the accuracy of my GPS position?”**



01

# The Ginan Software Toolkit

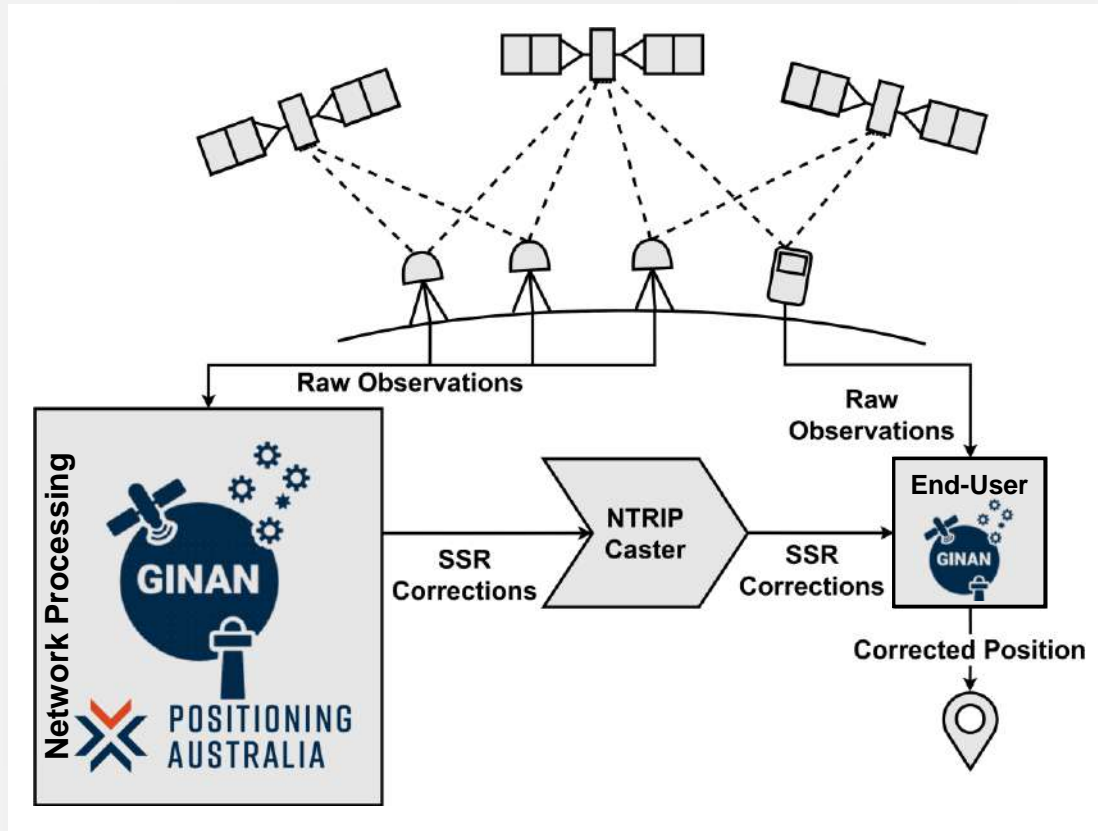
A Brief Overview



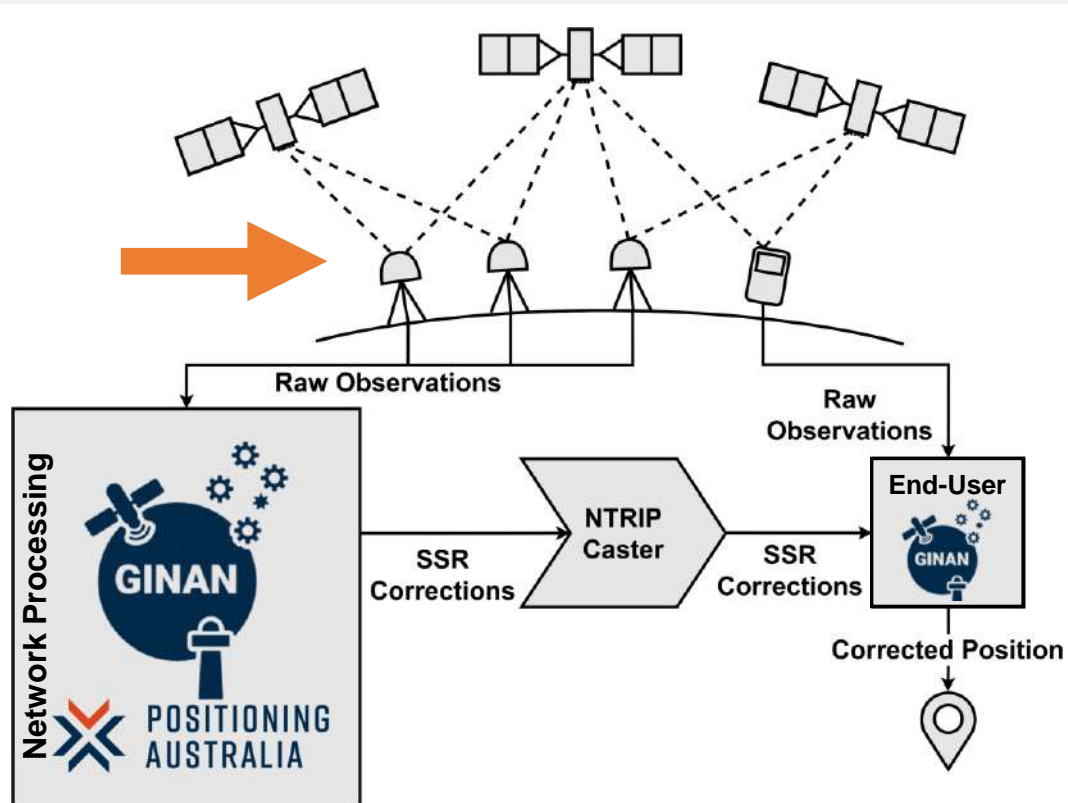
The Industrial  
Sciences Group



# Ginan – Overview

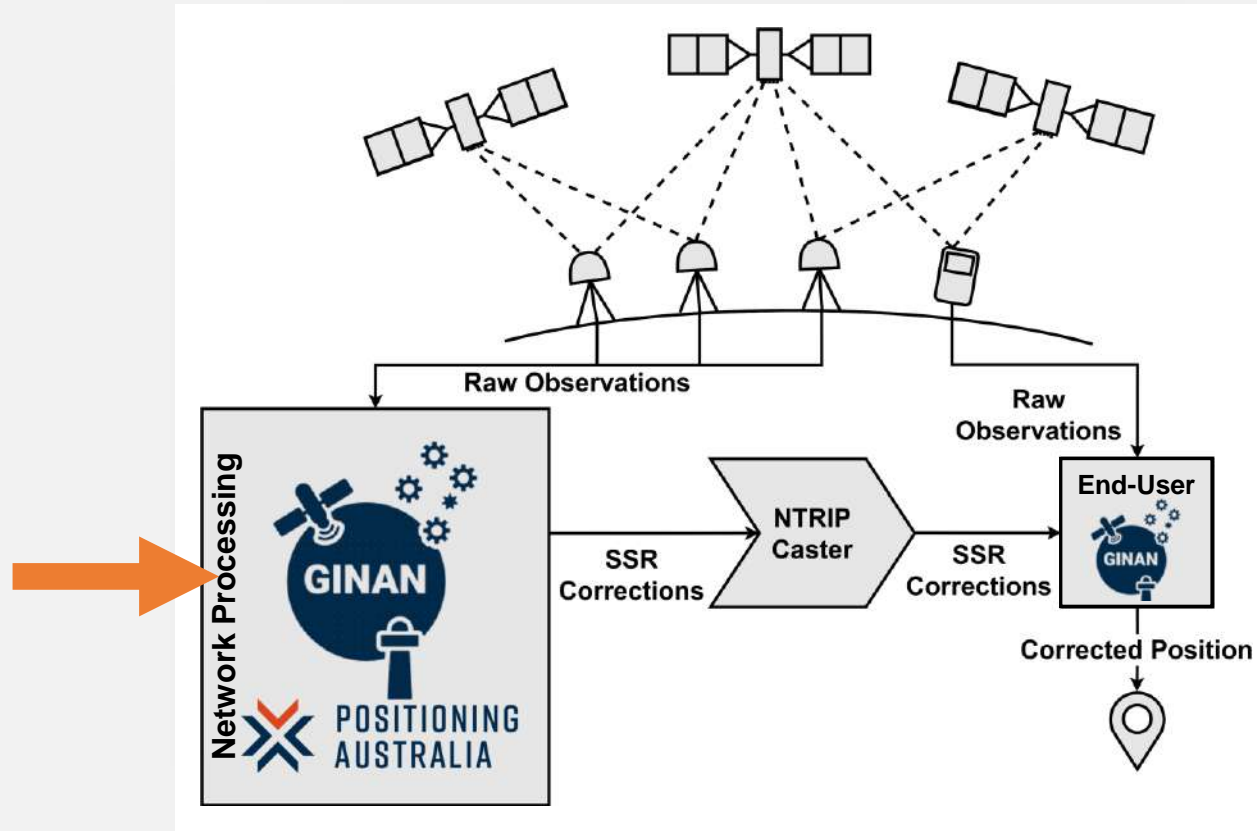


# Ginan – Overview

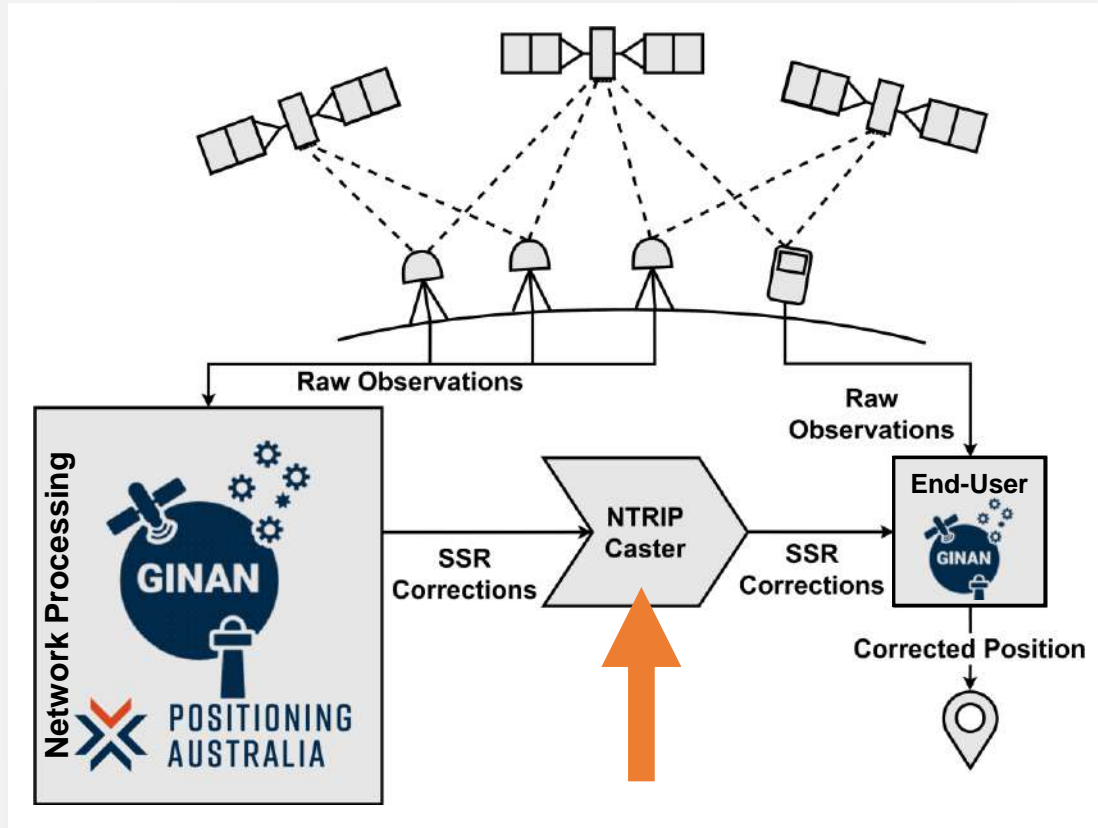




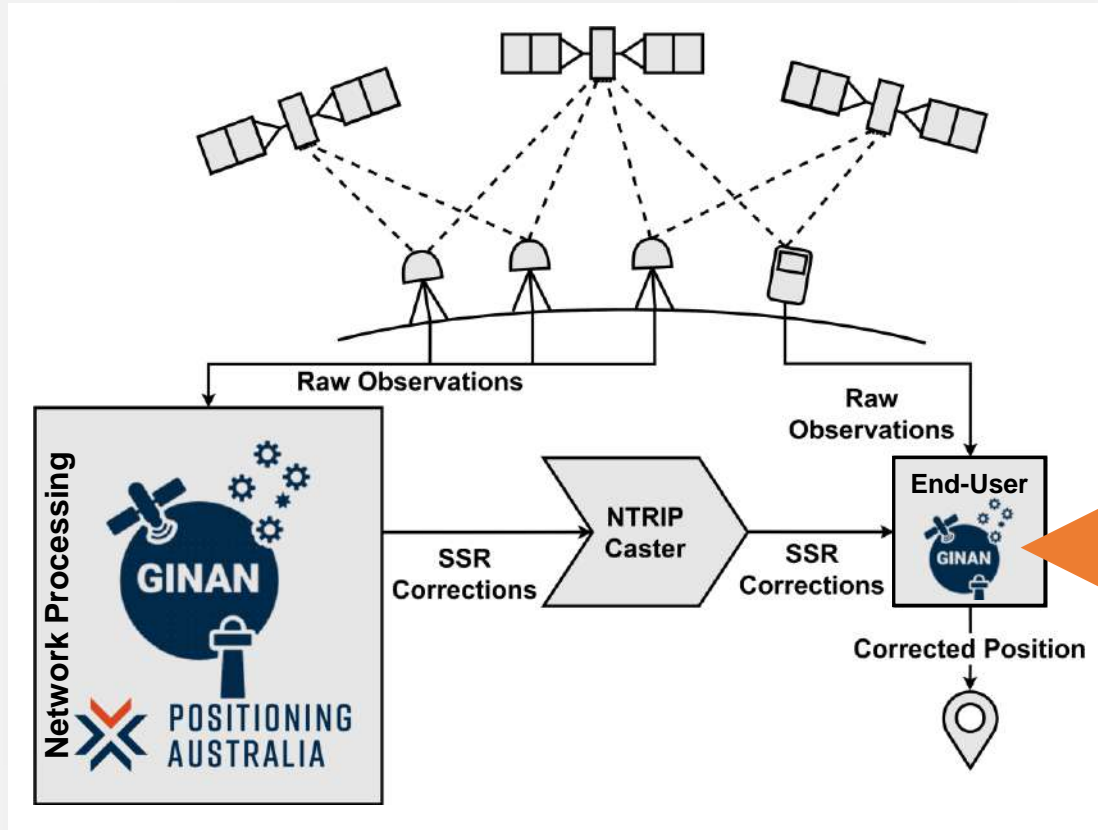
# Ginan – Overview



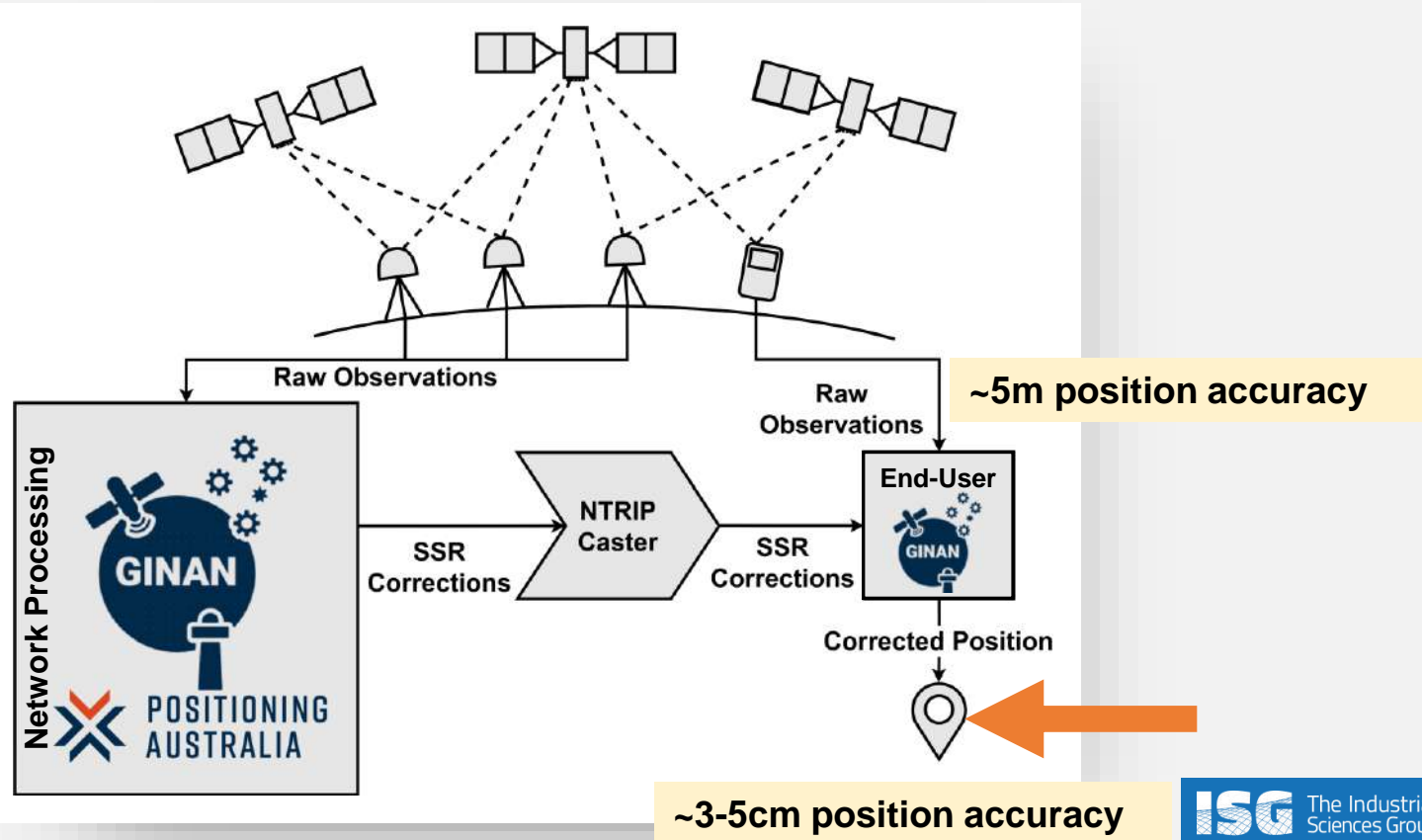
# Ginan – Overview



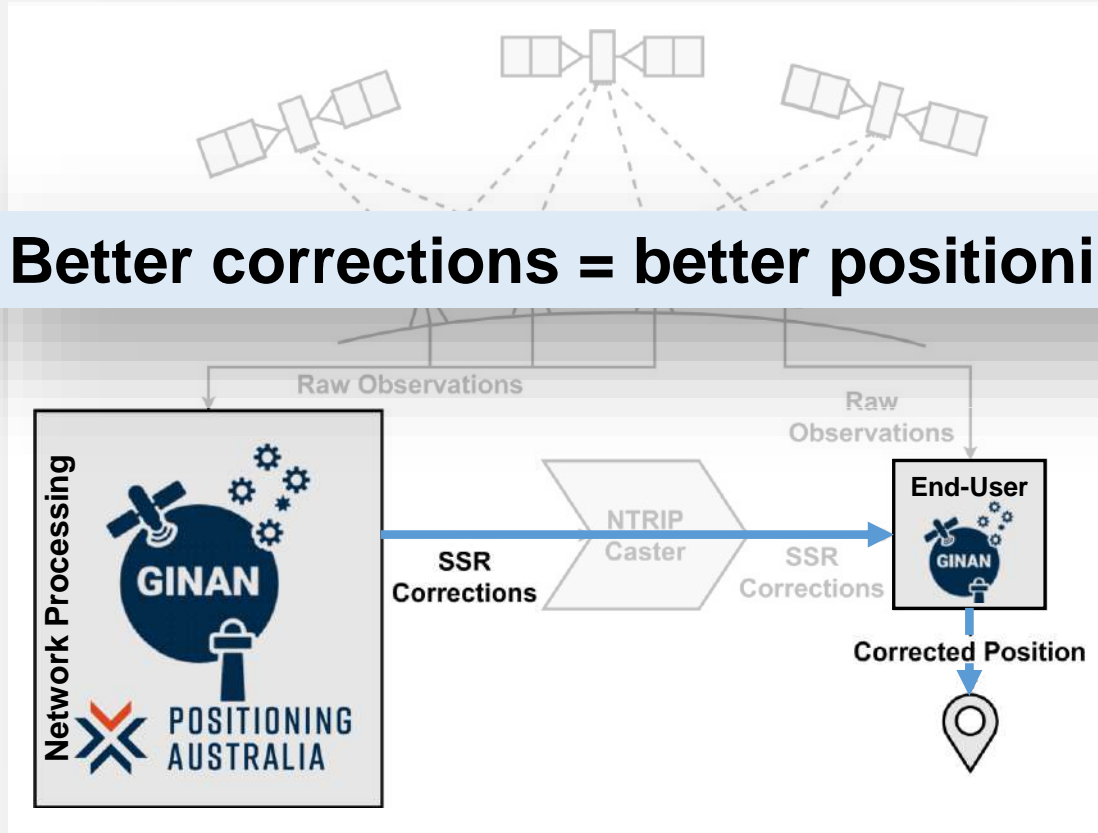
# Ginan – Overview



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# Ginan – Overview



# Ginan – ISG Contributions

- **Smoothing** – RTS, fixed-lag
- **State-transition modelling** – clock rate & acceleration, First-order Gauss-Markov
- **Filter stability** – non-PD covariance reshaping, Joseph stabilisation
- **Outlier detection** – time-series, Chi-squared, cycle slip detection + repair
- Processing **satellite laser ranging** data
- ...

02

# Smoothing

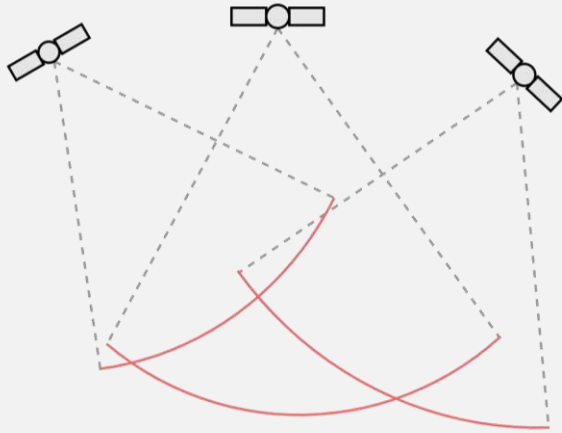
The Benefit of Hindsight



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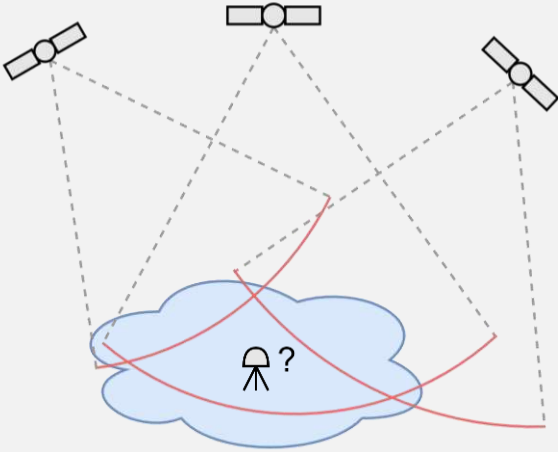


# Smoothing – An Example



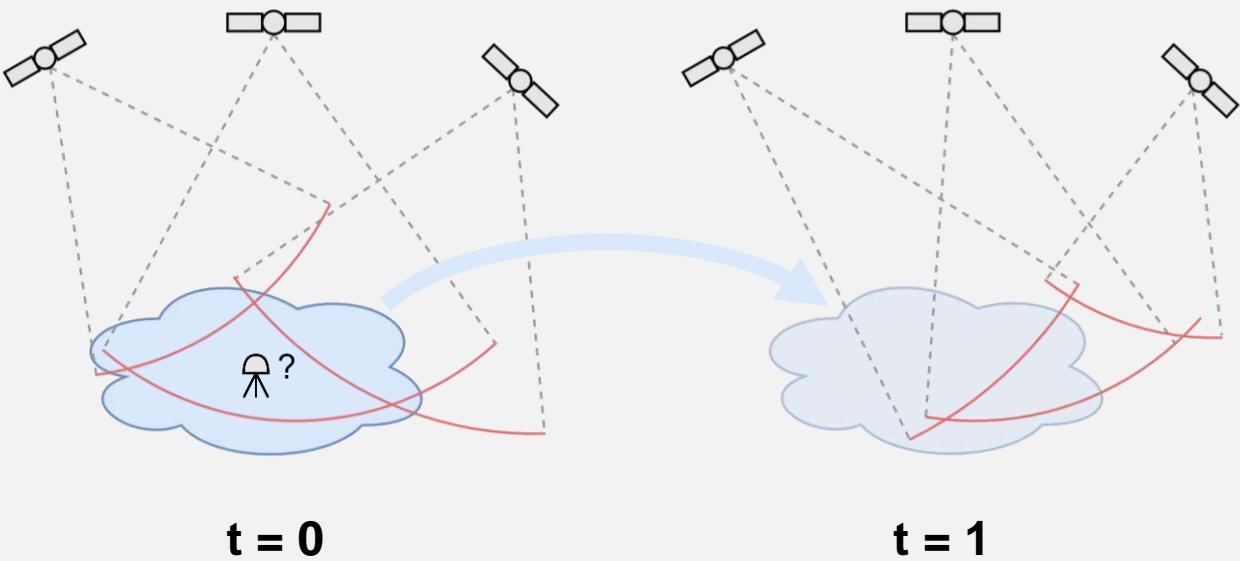


# Smoothing – An Example

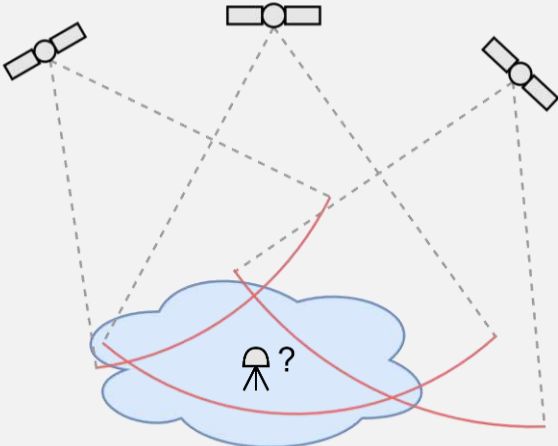


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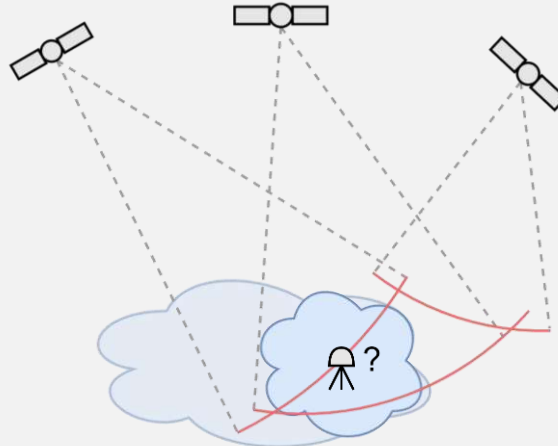
# Smoothing – An Example



# Smoothing – An Example

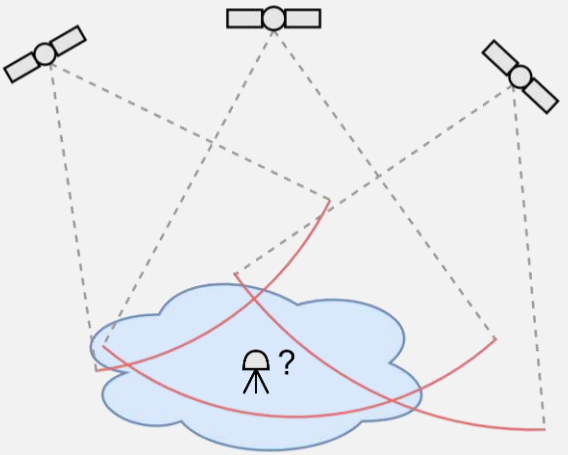


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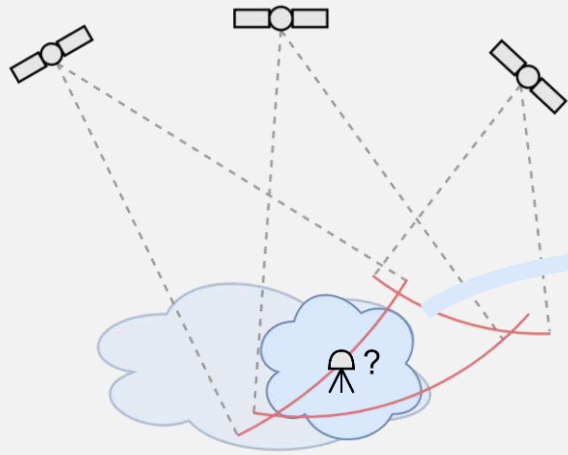


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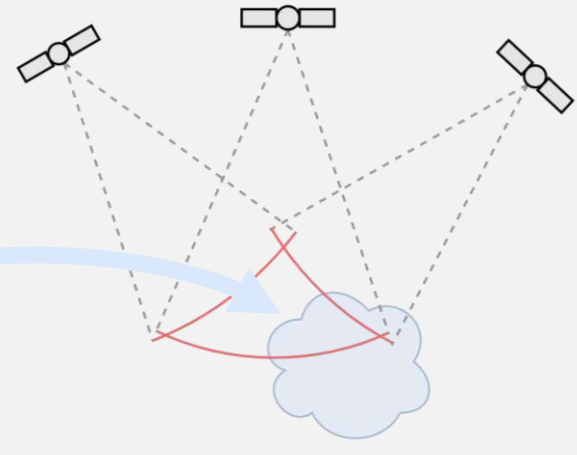
# Smoothing – An Example



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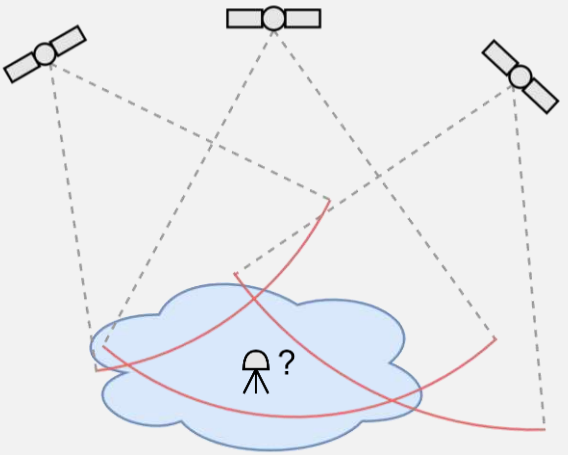


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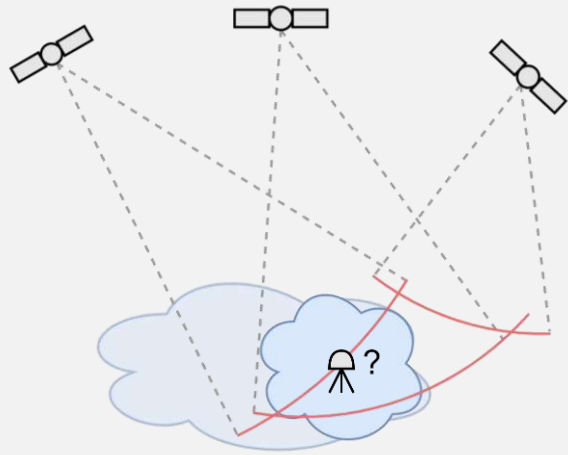


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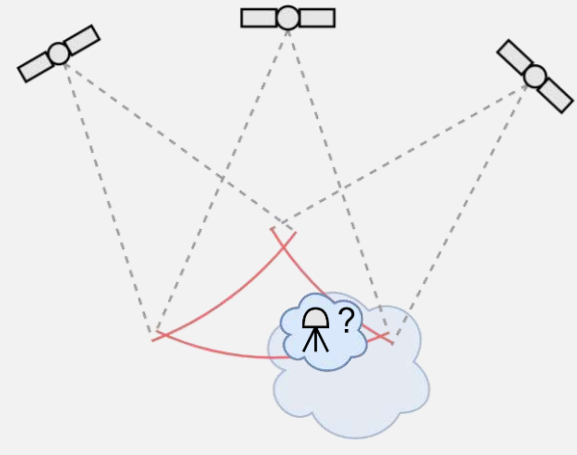
# Smoothing – An Example



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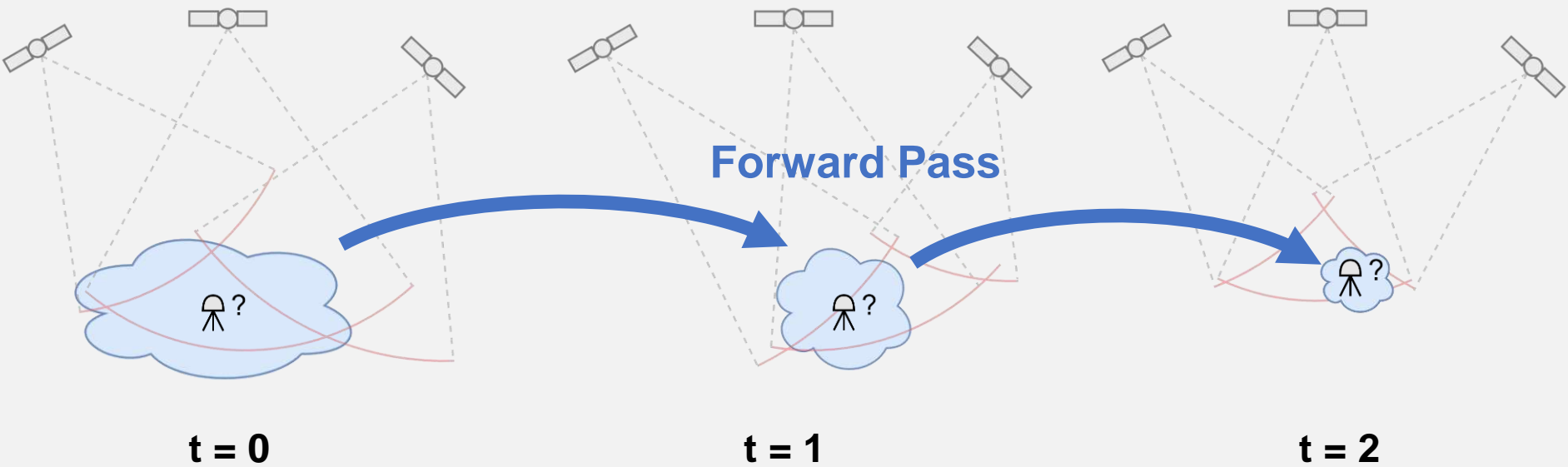


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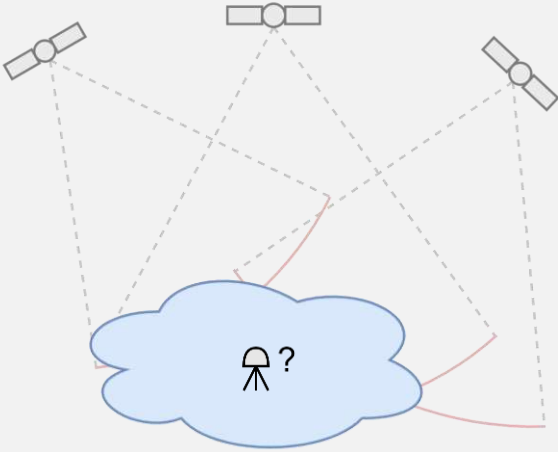


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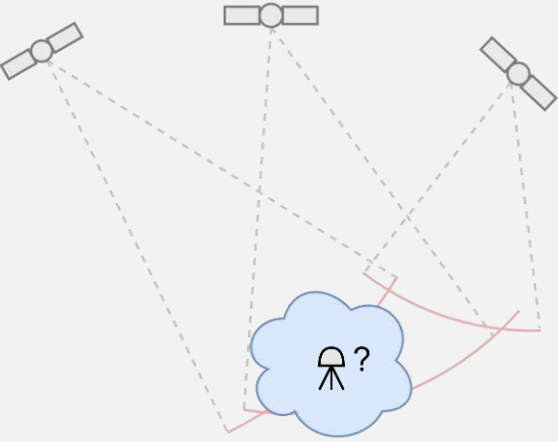
# Smoothing – An Example



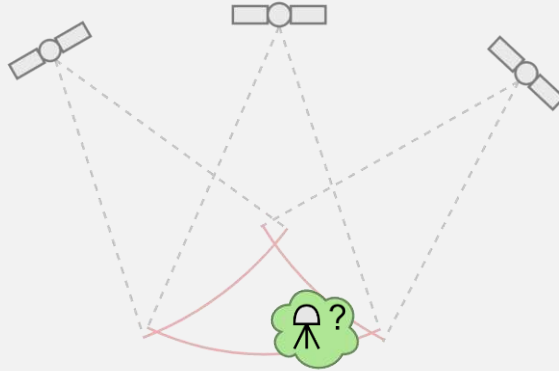
# Smoothing – An Example



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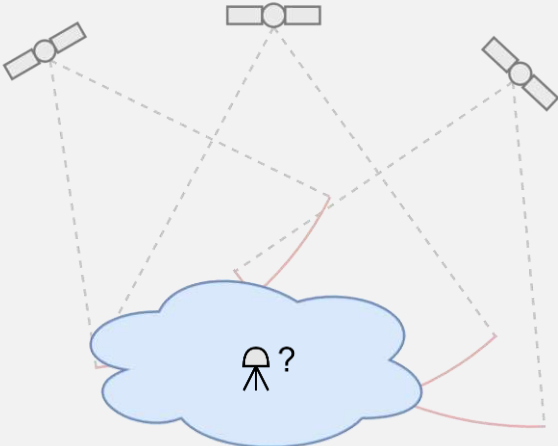


**t = 1**

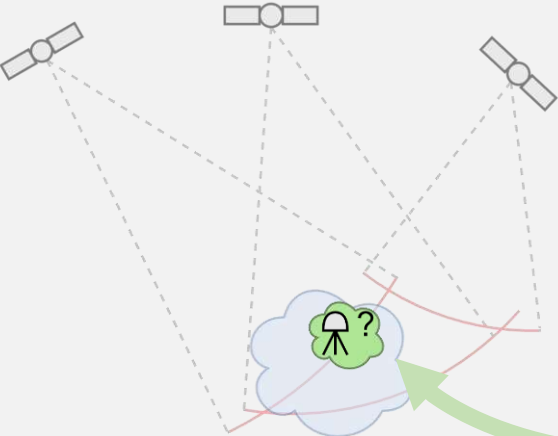


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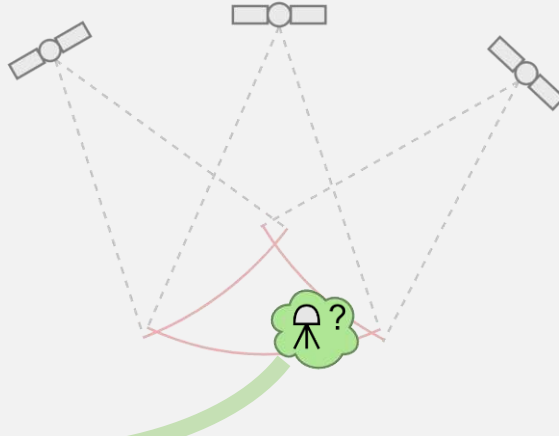
# Smoothing – An Example



**t = 0**



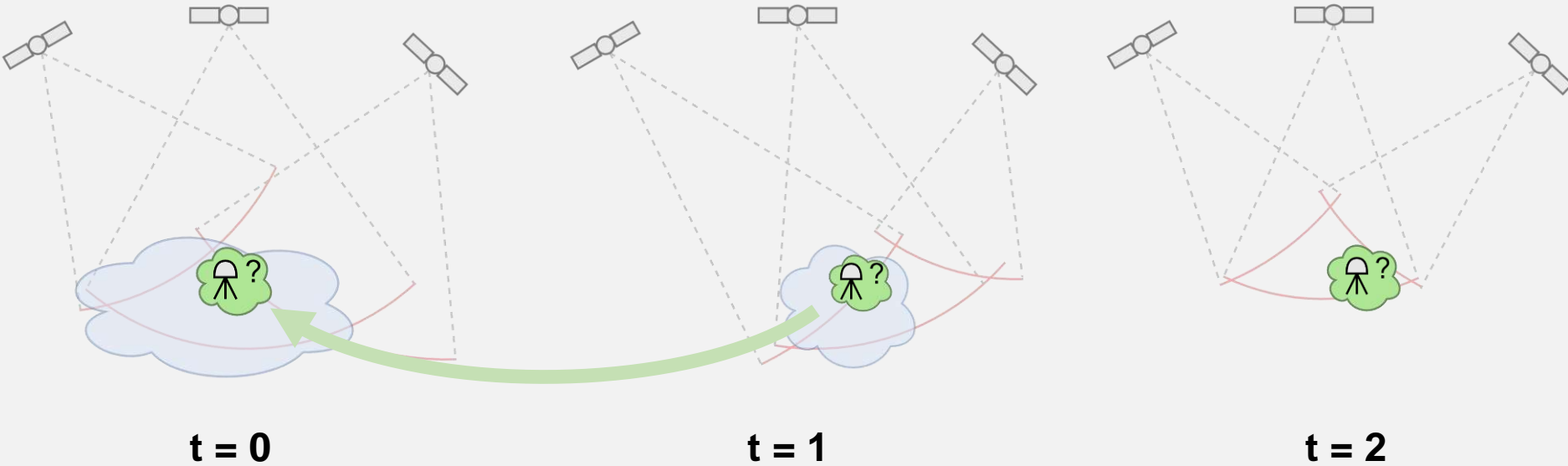
**t = 1**



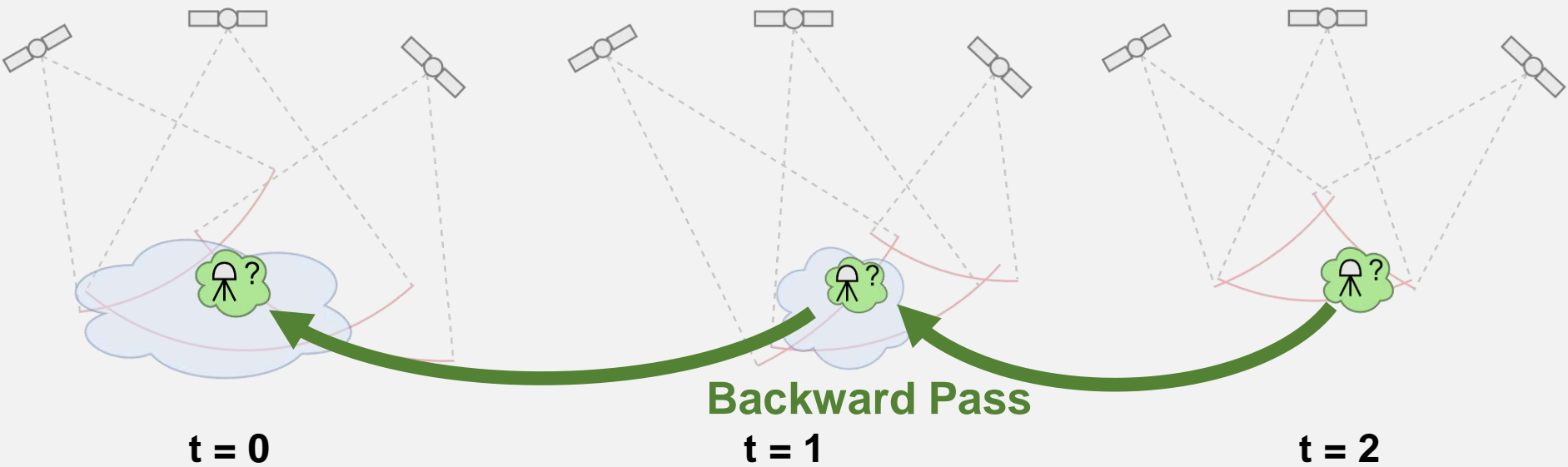
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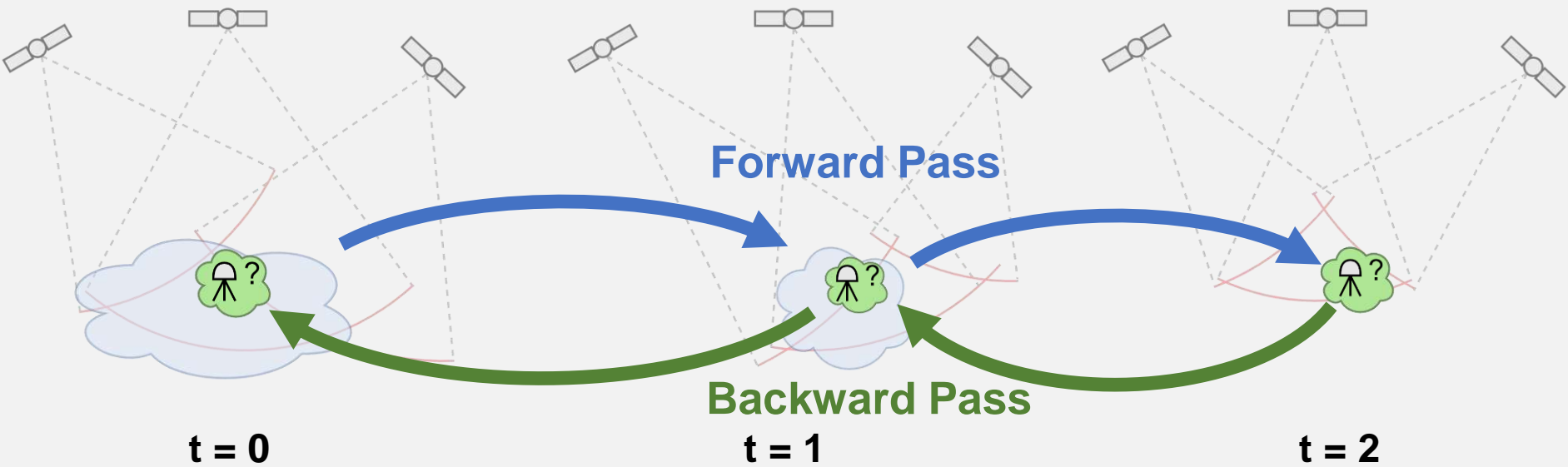
# Smoothing – An Example



# Smoothing – An Example



# Smoothing – An Example



# Smoothing – An Example

**‘Smoothing’ – an ‘a posteriori’ form of estimation  
(estimation using hindsight)**

t = 0

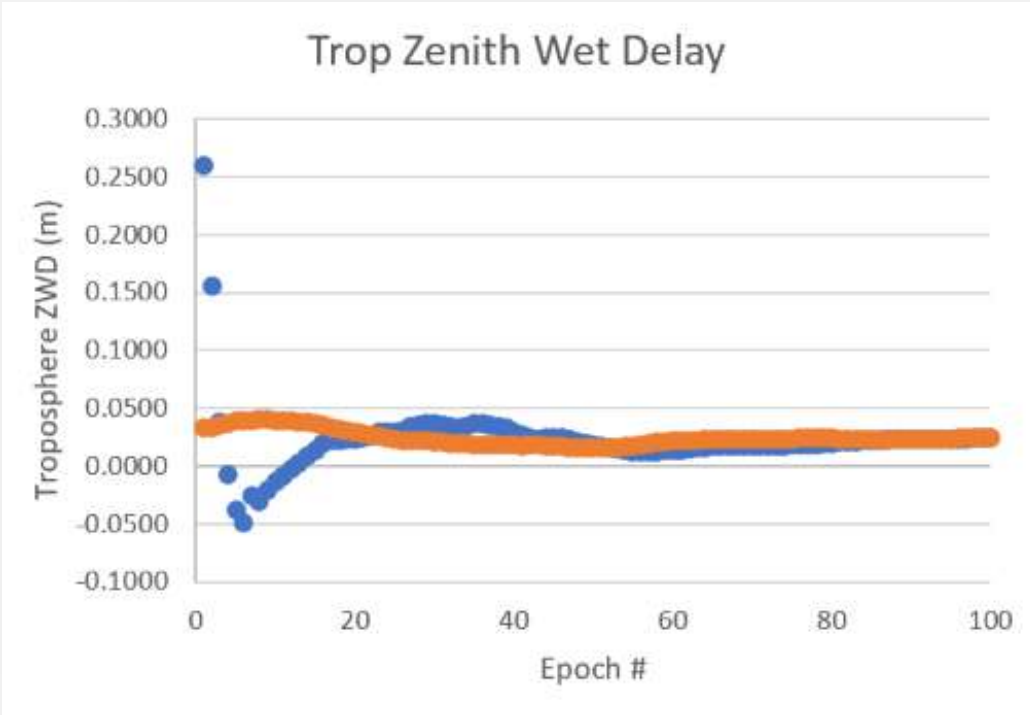
Backward Pass

t = 1

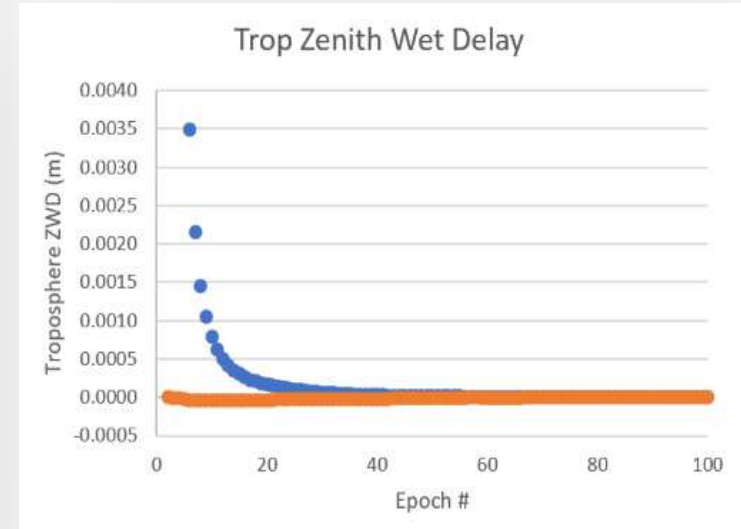
t = 2

# Smoothing in Ginan

Forward Pass  
Backward Pass



State Estimate

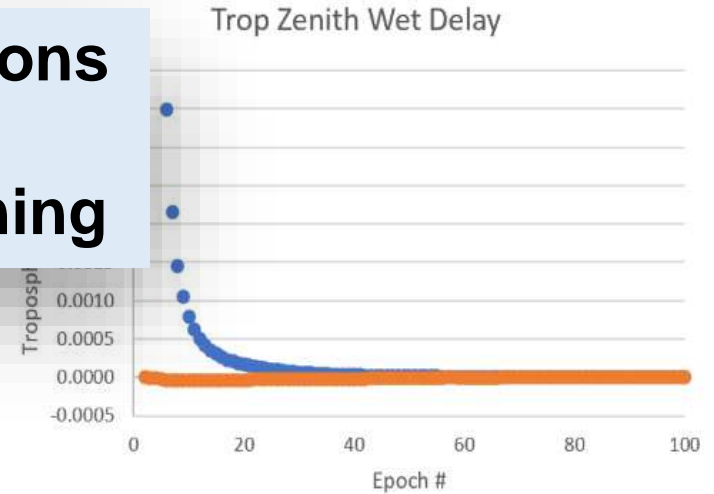
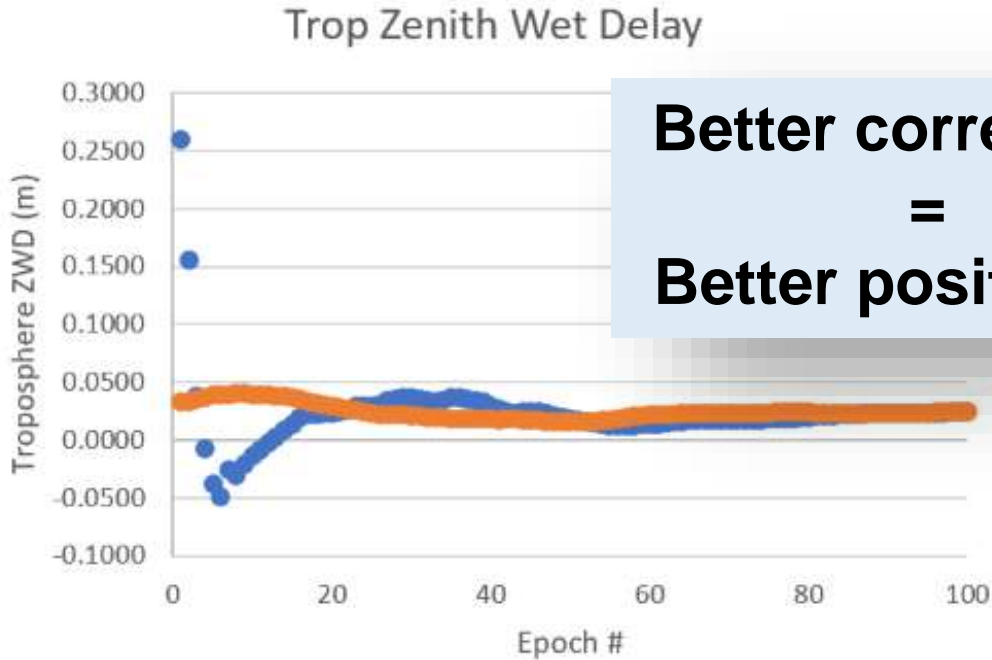


Variance

# Smoothing in Ginan

Forward Pass  
Backward Pass

**Better corrections  
=  
Better positioning**



**Variance**

**State Estimate**

# 03 First-Order Gauss-Markov Modelling

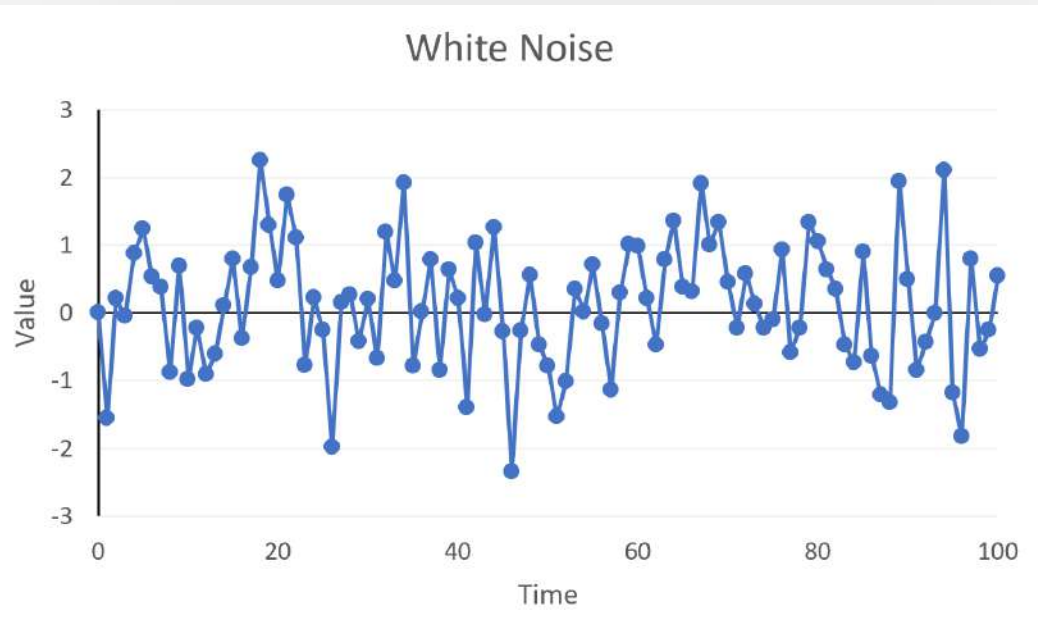
Using the Right Model



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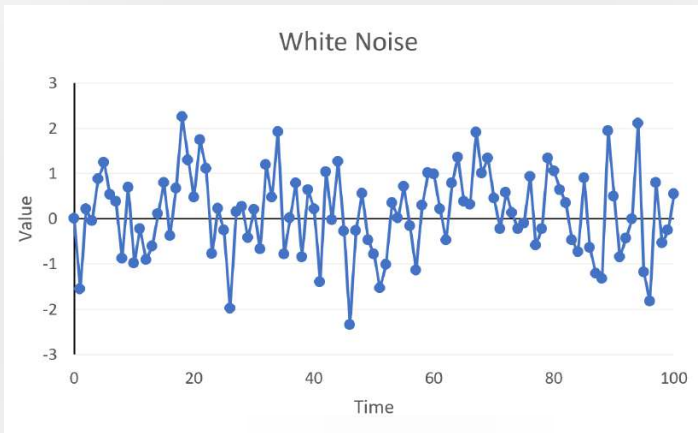
# White Noise to Random Walk



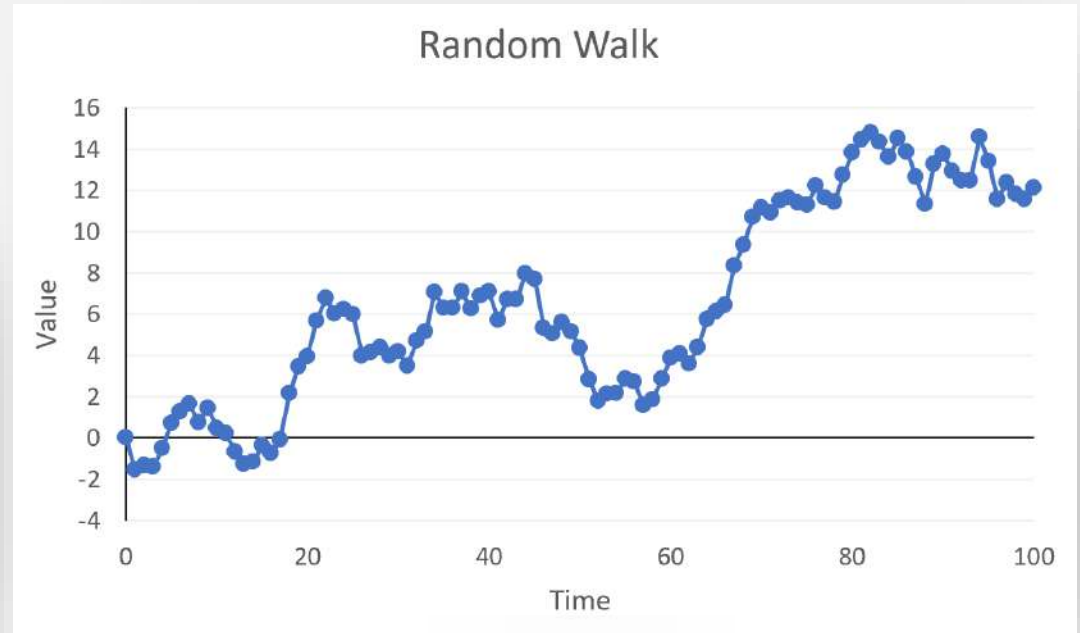
**Zero correlation**  
**Zero-mean**



# White Noise to Random Walk

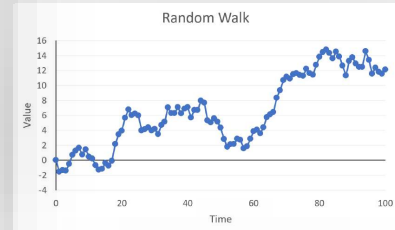
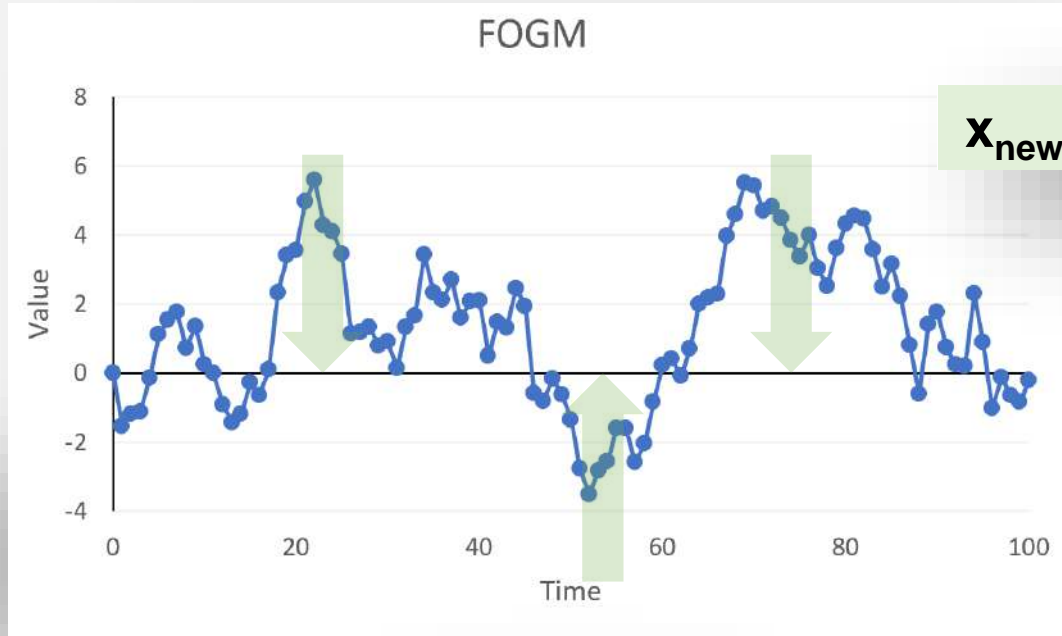
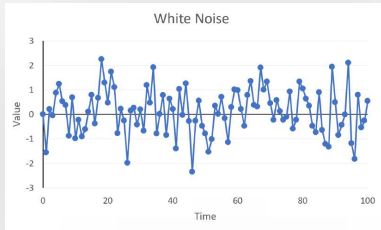


**Zero correlation**  
**Zero-mean**



**$\infty$  correlation**  
**No mean**

# White Noise to Random Walk



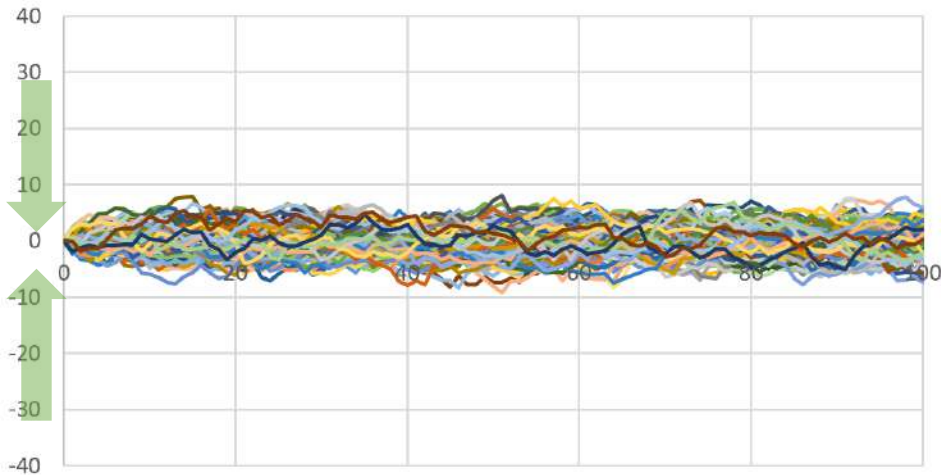
Zero correlation  
Zero-mean

Finite correlation  
Zero-mean

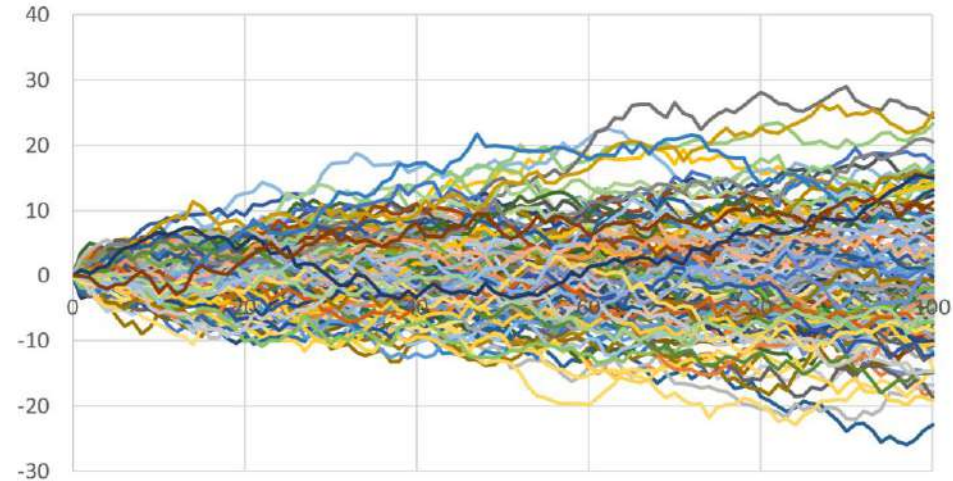
$\infty$  correlation  
No mean

# White Noise to Random Walk

FOGM



Random Walk



# FOGM modelling in Ginan

- [Graph of FOGM results]

**Better corrections  
=  
Better positioning**

# Conclusions

Better corrections = Better positioning

# Conclusions

- Outcomes
- [???

**Better corrections = Better positioning**

# Questions





# Appendix



# RTS Smoothing Equations

$$\hat{\mathbf{x}}_{k|n} = \hat{\mathbf{x}}_{k|k} + \mathbf{C}_k \left( \hat{\mathbf{x}}_{k+1|n} - \hat{\mathbf{x}}_{k+1|k} \right)$$
$$\mathbf{P}_{k|n} = \mathbf{P}_{k|k} + \mathbf{C}_k \left( \mathbf{P}_{k+1|n} - \mathbf{P}_{k+1|k} \right) \mathbf{C}_k^T$$

where

$$\mathbf{C}_k = \mathbf{P}_{k|k} \mathbf{F}_{k+1}^T \mathbf{P}_{k+1|k}^{-1}$$

# RTS Smoothing Equations

$$\hat{\mathbf{x}}_{k|n} = \hat{\mathbf{x}}_{k|k} + \mathbf{C}_k \left( \hat{\mathbf{x}}_{k+1|n} - \hat{\mathbf{x}}_{k+1|k} \right)$$
$$\mathbf{P}_{k|n} = \mathbf{P}_{k|k} + \mathbf{C}_k \left( \mathbf{P}_{k+1|n} - \mathbf{P}_{k+1|k} \right) \mathbf{C}_k^T$$

where

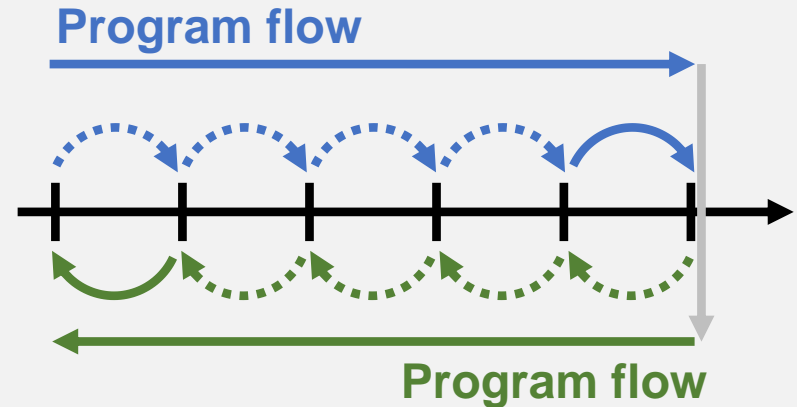
$$\mathbf{C}_k = \mathbf{P}_{k|k} \mathbf{F}_{k+1}^T \mathbf{P}_{k+1|k}^{-1}$$

More process noise =

- Larger  $\mathbf{P}_{k+1|k}$
- Smaller  $\mathbf{P}_{k+1|k}^{-1}$
- Smaller  $\mathbf{C}_k$
- Smaller impact of future states

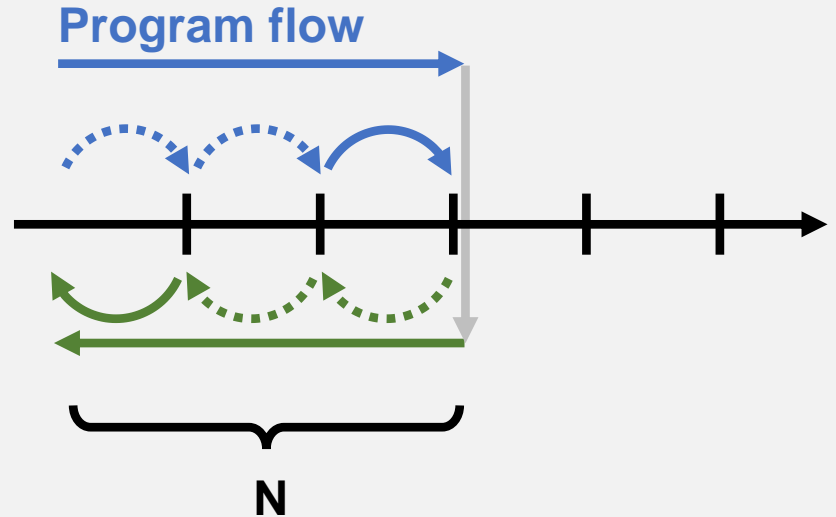
# Fixed-Interval Smoothing (RTS)

- Batch-processes data within a fixed time interval (e.g. daily)
- Improves estimation accuracy across whole interval, particularly towards the start of run



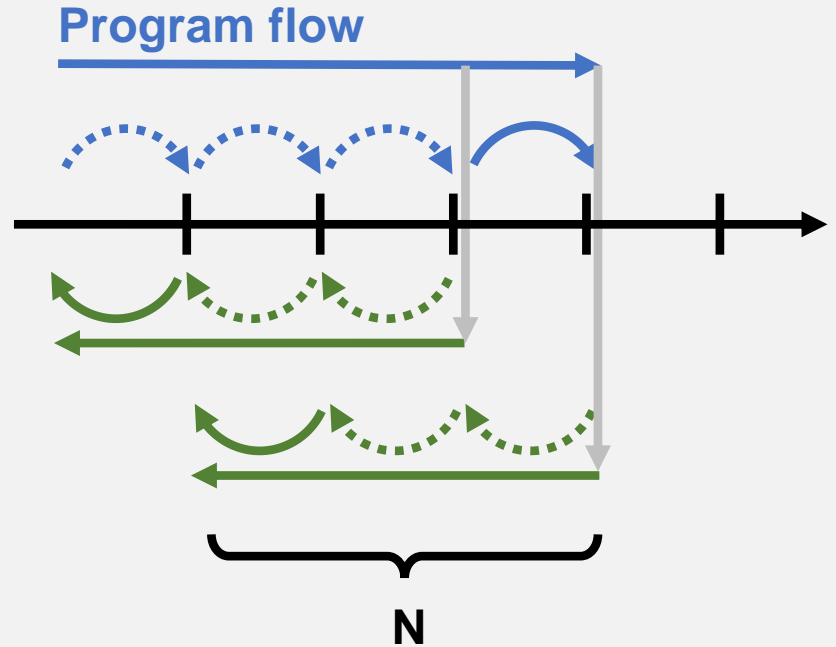
# Fixed-lag Smoothing

- Processes data in near-real-time, depending on lag  $N$
- Improves estimation accuracy within the last  $N$  timesteps



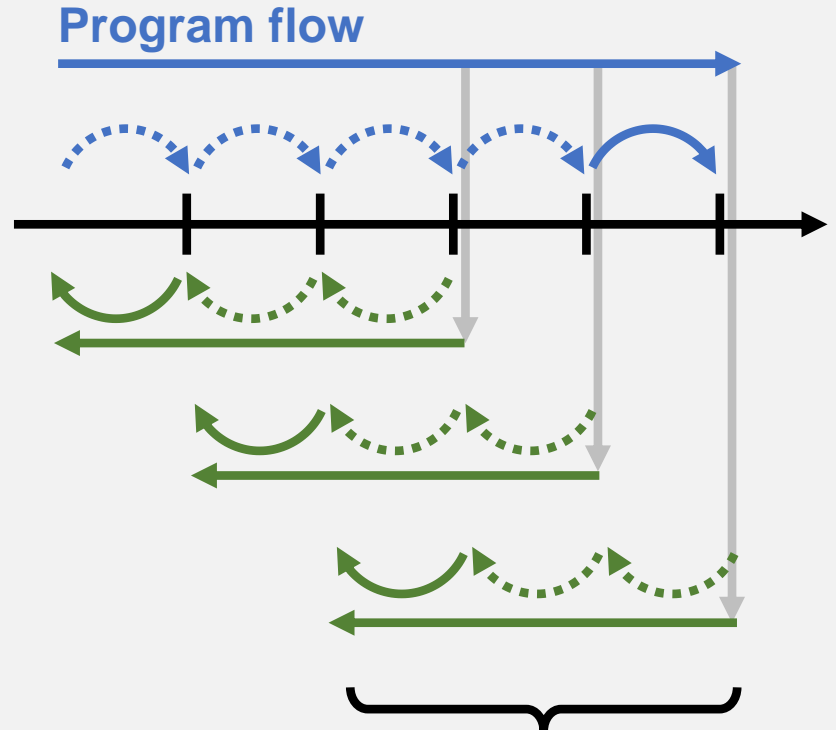
# Fixed-lag Smoothing

- Processes data in near-real-time, depending on lag  $N$
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# Fixed-lag Smoothing

- Processes data in near-real-time, depending on lag  $N$
- Improves estimation accuracy within the last  $N$  timesteps



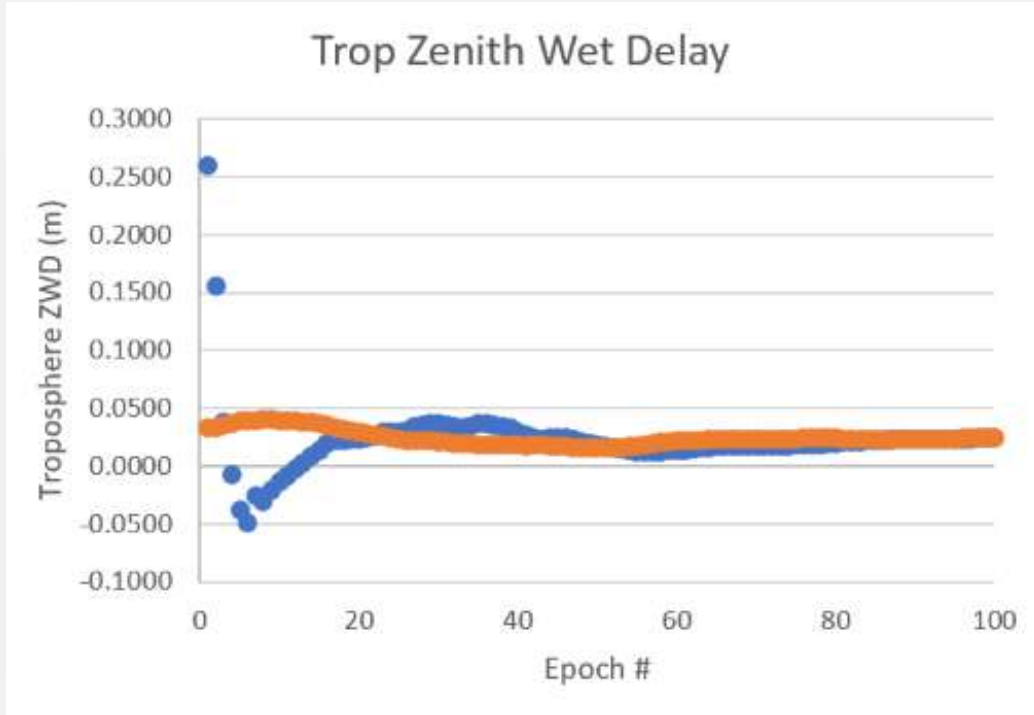


[replace w position?]

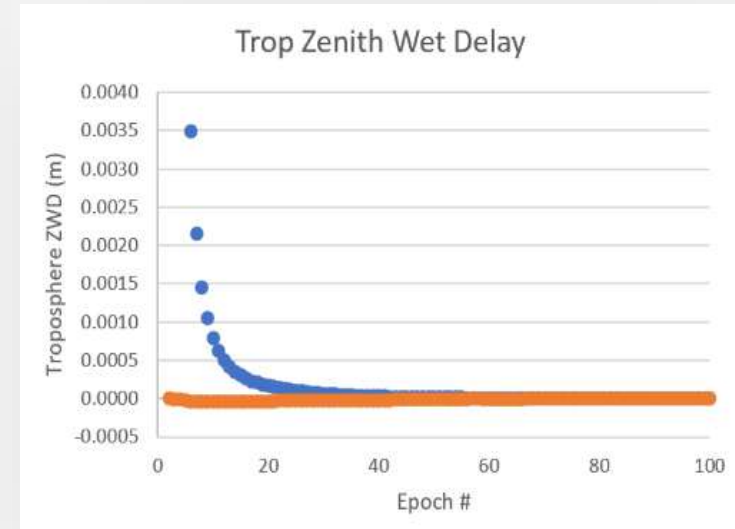
# Smoothing in Ginan

Forward Pass

Backward Pass



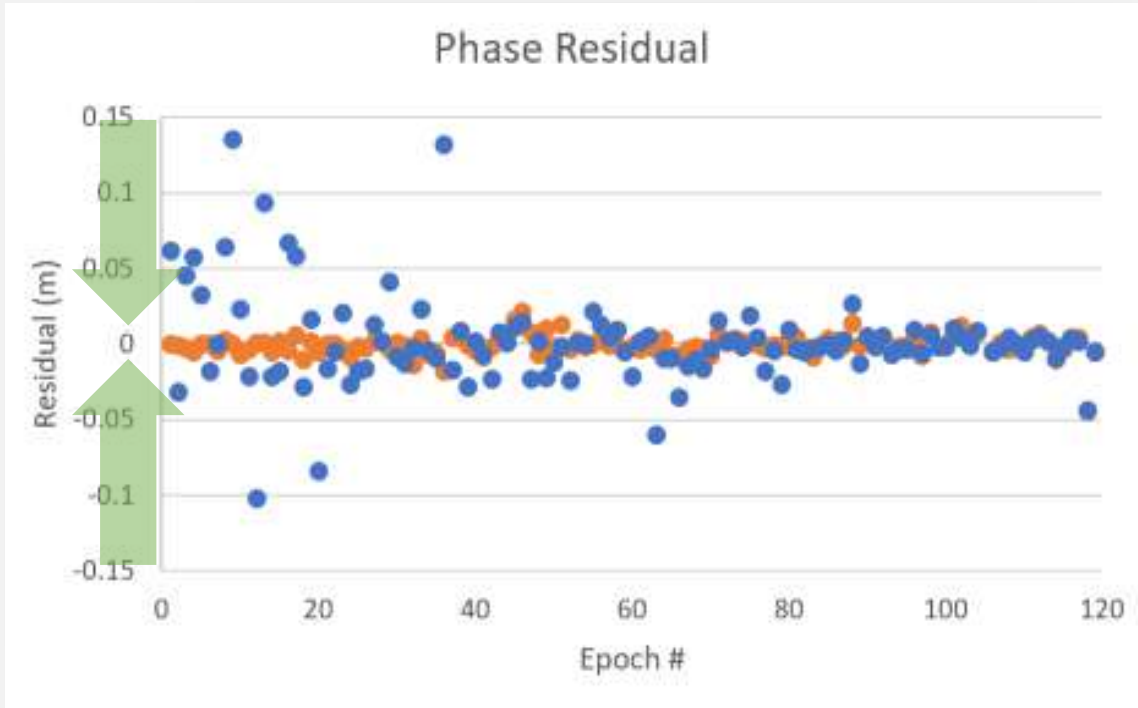
State Estimate



Variance

# Smoothing in Ginan

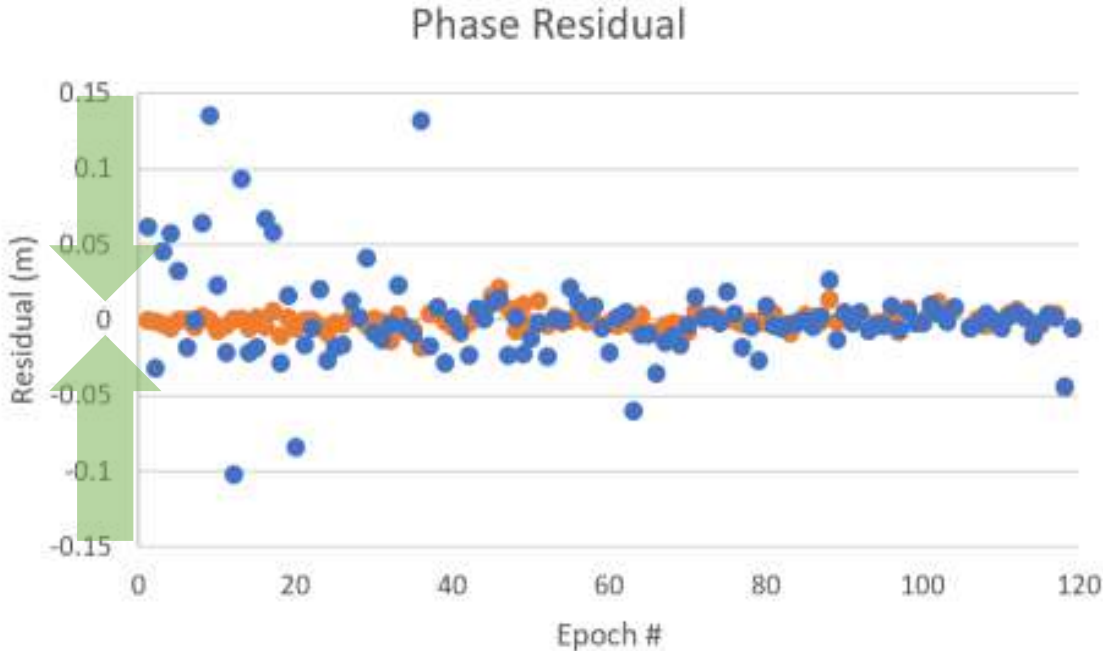
Forward Pass  
Backward Pass



**Observation Residuals**

# Smoothing in Ginan

Forward Pass  
Backward Pass



**Better corrections**  
=  
**Better positioning**

**Observation Residuals**