Maths matters

Industrial mathematics: here and now . . . positive in all directions

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In a recent edition of the *Gazette*, Graeme Wake [1] wrote of the success of industrial mathematics as something that, like the crest of a wave, is about to 'break through'. In this article I would like to inform AustMS members of projects already underway in industrial mathematics and statistics through MASCOS (the Australian Research Council's Centre of Excellence for Mathematics and Statistics of Complex Systems). To extend the nautical metaphor — we are surfing down a wave now, and there are more waves coming!

Since 2008 MASCOS has conducted 14 projects in industry. Industry sectors include transport (NSW Roads and Traffic Authority, and Vicroads), defence (Defence Science and Technology Organisation), coal mining (MSEC Consulting Group), medical devices (Cochlear), mental health (the Mental Health Research Institute) and nuclear science (ANSTO). For project details see MASCOS annual reports at www.complex.org.au.

MASCOS projects are not like typical consulting projects, where a specific problem is solved using existing mathematics, and recommendations are made. Rather, each project starts with an open problem set by the client. For example 'design a new traffic control system to reduce congestion', or 'design a statistical model to predict the cost of road network simulations based on network complexity' or 'propose a new theoretical framework to improve the confidence in risk modelling of ground movement due to underground mining'. These projects require original applied research in a combination of mathematics, statistics and engineering to fill the gap identified by the open problem. Hence, in addition to the commercial value to the industry client, each project has research value to the professional mathematician or statistician. Research areas covered include statistical mechanics of non-equilibrium systems, extreme value theory, classification of high-dimensional data, risk modelling and more.

All these projects have been fully funded by the industrial clients themselves. Unlike OCCAM in the UK (Oxford Centre for Collaborative Applied Mathematics), mentioned in [1], we have not had to rely on Saudi Arabian funding.

There are clear reasons for the successes so far, and yet there are barriers to further success. Contrary to popular opinion, these barriers are not due to lack of government funding or to private industry or university administrations. Rather

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the barriers lie within the mathematics community itself, such as some cultural attitudes partly related to the academic promotion process (see 'Barriers' below).

We now give three sample projects as illustrations.

Project 1.
Traffic networks: the dynamics of congestion

The transport sector, in particular road traffic modelling, has been the sector of greatest activity for MASCOS projects. It has been a perfect combination of meeting an urgent industry need (how to reduce road traffic congestion without building new infrastructure) and research interest (improving the methods of modelling non-equilibrium systems that are constrained in a network structure). This was verbalised by a Vicroads manager who said at a meeting with MASCOS:

> We know traffic engineering very well. Now we want to hear from mathematicians and physicists to help solve the big problem of congestion.

In the two-stage project with Vicroads 'Arterial road congestion: network modelling and improved control', researchers from the Critical Phenomena group of the MASCOS Melbourne University node, led by Dr Tim Garoni and Dr Jan de Gier, have developed a Cellular Automata (CA) simulation model of traffic flow in generic urban networks. The research will be published in 2010 in *Physical Review* (see [2]).

The model was applied to a specific road network in the Melbourne suburb of Kew, the objective being to develop alternative traffic light control strategies for the urban road networks managed by Vicroads. Currently, the only input data for the signal control systems is provided from induction-loop detectors, and this information is rather limited. MASCOS's CA model was used to study the performance of a range of more general adaptive traffic signal systems, which utilise more detailed input data. The model, and its results, are being used as support for upgrading to new traffic detection systems for Melbourne roads.

In particular, the CA model was used to study the relative efficiencies of two distinct types of adaptive traffic signal systems; a system that only considers the congestion of upstream links, versus a system that considers the congestion of both upstream and downstream links. The simulations suggested that the latter system is more efficient — around 5% better in the case of the Kew network. A 5% reduction in congestion for no extra infrastructure cost is highly significant.

The project has raised a wide range of scientific issues. In contrast to traffic on freeways, traffic flow on networks is, as yet, poorly understood. The CA model and its application have led to the following fundamental issues being addressed in the ongoing project [2]:

- study of parameter sensitivity and identification of critical traffic states
- determination of phase diagrams and phase structures, that is, what are the fundamentally different behaviours of traffic on a network?

- identification of critical-length scales such as mean free paths and correlation lengths
- investigation of the existence of scaling
- investigation of non-equilibrium work relations and fluctuation theorems: can we describe global states of traffic using thermodynamic quantities, and how do these relate to fluctuations in density and flow?
- development of a computationally effective traffic model on a network with about 100 intersections. This requires identification of relevant features, so that irrelevant details can be neglected.
- correlations between optimisation functions: are Minimal Delay, Optimal Flow, and Total Travel Time equivalent measures?

Project 2.
Guidelines for designing and analysing traffic micro-simulations

The expense of designing and building new road infrastructure or testing alternative traffic scenarios can run to billions of dollars. Therefore, all design changes are first assessed using traffic micro-simulation software.

In 2009, MASCOS identified a need for rigorous statistical analysis of the outputs of these simulations. In 2010 MASCOS is nearing completion of a three-stage project with the NSW Road and Transport Authority's Network Performance Development Group to design and build a rigorous statistical framework to analyse the outputs of traffic micro-simulations. The project is being conducted by the author, on behalf of MASCOS's UNSW node, assisted by Dr Sandy Clarke of Melbourne University's Statistical Consulting Centre.

Exploratory Data Analysis (EDA) techniques, traditionally used in industrial quality control, have been adapted to the analysis and design of traffic micro-simulations. EDA techniques have been used to gain insight into the salient features of the output of a wide range of traffic simulations, ranging from small arterial networks to freeways and entire suburbs. The EDA has shown, for example, the importance of extreme events or 'outliers'. Outliers are being used as a diagnostic tool when correlated with other inputs to distinguish between model errors and a real occurrence of a rare event (for example, a major accident).

Once high-quality simulation data is obtained, the sources of variability in the simulations are considered, using ANOVA, as well as the implications of this for the precision of estimates of network characteristics, such as vehicle hours travelled. This informs the choice of run size and comparisons between different traffic scenarios.

The final stage of the project will seek to find functional relationships between network features, output precision, the number of simulation runs and the complexity of the network being simulated. Network complexity is described by features such as the number of zones, number of road links per intersection, size and shape of the network, total number of vehicle trips, time duration of simulation, and boundary congestion effects (the effect of delayed or 'unreleased' vehicles that could not enter the network due to congestion or incomplete trips, and of vehicles that could not exit the network due to congestion).

The MASCOS/RTA project is unique in its application of advanced statistical methods to traffic micro-simulation. It has aroused great interest in the RTA and the wider traffic-modelling community. A technical paper and special session on its applications to policy evaluations in transport will be presented at the 17th World Congress on Intelligent Transport Systems in Busan, Korea in October 2010 (see [3]).

Project 3.
Mining and geo-mechanics

MASCOS has established a three-stage project with MSEC Pty Ltd (Mine Subsidence Engineering Consultants) of Sydney, to develop statistical methods using Extreme Value Theory (EVT) in order to improve prediction of the magnitude of ground subsidence due to underground coal mining, and the consequential impacts on structures (see [4]).

The first stage of this work was completed by Dr Scott Sisson of the School of Mathematics and Statistics, UNSW. This involved exploratory statistical analyses to quantify the probability that a future ground strain caused by mining exceeds a specified maximum tolerable subsidence (that is, a trigger point). It was demonstrated that using EVT-motivated models to describe the extreme tails of MSEC's observed strain data resulted in more credible fits than those based on alternative models originating from the full dataset. As a consequence, the predictions of future extreme subsidence in excess of the trigger points are more reliable.

The second stage of this project is being conducted by MASCOS post-doc Dr Yao-ban Chan, who will develop the statistical models more precisely. This involves the use of regression methods to improve accuracy and precision by the inclusion of relevant explanatory variables, such as the distance from the point of interest to the mine (a 'far field' analysis), and the modelling of the relationship of subsidence strain and curvature. Dr Chan is also simplifying the implementation of the EVT methods by supplying MSEC with programs written in the 'R' software package, with documentation for easy application.

In 2010, work on the third and final stage of the project will aim to incorporate the effect of multiple 'longwalls' (the mines excavated by drilling equipment moving underground), as well as smoothing raw curvature data. The application of statistical EVT to predict ground subsidence is a new application of the theory; it is also the first time that state-of-the-art statistics has been used in this particular industry sector [3].

The project hopes to establish, in a statistically rigorous manner, the extent to which factors (such as geology, valley width, distance to the leading edge of the longwall) known to influence ground movements in general, primarily drive the process of extreme strains or 'upsidence' (upwards movement of a valley floor), and the extent to which these can then be used to predict future ground maximum movements at new locations.

The results from Stage 1 were used in MSEC's 2009 submissions to the NSW Government. Stage 2 and 3 results will assist MSEC in its consultancy advice to Government and mining companies on the effects of proposed underground mines.

Reasons for success

By analysing the most successful projects, and how they evolved, we can discern some characteristics that may contribute to the success of industrial mathematics projects in general.

Firstly the majority of the large-scale projects were created by the MASCOS industry division itself. We did not wait for a company to come to us with a problem. Rather, MASCOS approached an industry sector with the general outline of a project. For example, the idea of applying the methods of statistical mechanics and critical phenomena to traffic flow dynamics was presented to the traffic management divisions of the RTA and Vicroads.

By taking this proactive approach of targeting industry sectors for specific projects we are able to satisfy the 'double' demand of commercial benefit to the industry partner and genuine research value for the mathematician. Furthermore this approach allowed us a significant role in shaping the project structure with respect to duration (all projects span a minimum of six months) and skill level (PhD level and above).

A second feature contributing to success was the researchers' familiarisation with the domain of the problem at hand. Researchers on traffic projects learnt the basic elements of intersection control systems, traffic engineering principles and terminology. Researchers on the ANSTO project became familiar with the basics of nuclear research reactor operation, control and safety systems, regulations, and instrument calibration requirements. A mathematical modelling project that is devoid of such engineering and technology content would be of dubious value.

Project familiarisation was coupled with a willingness to 'get one's hands dirty' with real data. Surprisingly, this has been an obstacle when staffing projects. Patience and a degree of worldliness are required to accept that real-world data is never like the 'toy' data presented in text books that students are trained on. However, that patience is repaid many times over with the intellectual satisfaction of subjecting theory to a reality test in a project that also makes a difference in the world.

Barriers

The single greatest barrier to further success in industrial projects has been a shortage of willing mathematicians and statisticians to participate in the projects.

The reasoning behind this reluctance usually goes as follows:

Objection 1: Industry-based projects are mere consulting and of little scientific value
Objection 2: Industry-based projects only lead to B-grade publications, at best
Conclusion: Working on an industry project is bad for my career advancement.

Earlier in this article I presented the scientific value of just three of our fourteen projects. Moreover, the history of mathematics and statistics is full of cases where work on a real-world problem led to a major advance in mathematics. Without the motivation and 'raw data' of the problem the theoretical advance may never have occurred. Here is a sample in chronological order.

- Euler initiated Graph Theory from his solution to the Koenigsberg bridge problem
- Gauss developed and demonstrated the Method of Least Squares as a way to predict the position of the asteroid Ceres
- Fourier developed what we know as 'Fourier analysis' from trying to solve the heat equation, which is of fundamental importance to all thermodynamics
- Heaviside step functions were developed to model electric current
- R.A. Fisher developed the Analysis of Variance and the entire basis of Designed Experiments to improve the efficiency of experiments on farming methods at Rothamsted Experimental Station
- Dantzig developed the Simplex Algorithm — the basis of linear programming and subsequent optimisation methods — as a way to solve very complex military scheduling problems that had arisen in World War II.

I trust no reader of this *Gazette* would claim that the careers of Gauss, Euler, Fourier, Heaviside, Fisher or Dantzig were degraded by the applied projects that resulted in their discoveries, nor that their work led to B-grade publications.

Projects in the real world can present a challenge to existing theory and that challenge can be idealised into new theory. Also, an industrial project may present as a novel application of existing theory. That is also a scientific contribution, as the use of new tools advances the domain area of the problem; see the traffic and mining examples above and [3] and [4]. In both cases the initial motivation came from a real physical problem.

There is a continuing strong demand for mathematicians and statisticians to perform applied research to solve industrial problems. At the very least this guarantees the mathematical sciences community a large and continuous source of serious problems to work on. What is required is some overcoming of false perceptions (within the mathematics community) on scientific value and publication prospects. Starting with a small project can often lead to bigger things. MASCOS researchers have two Linkage Grant applications submitted in 2010 that arose from industry projects.

Engagement with industry through successful projects brings a range of 'spin-off' benefits; for example, improved public recognition for the importance and value of the discipline. That in turn should enhance the view of the mathematical sciences in the eyes of many, including, most importantly, prospective students.

Finally, engagement with industry has one other major benefit — it reduces the dependence of the mathematical sciences on government funding (professional mathematicians' biggest source of complaint). So, engaging with industry really is positive in all directions!

Those interested in participating in industry-based projects that require skills in mathematics or statistics should contact the author at davids@complex.org.au and see www.complex.org.au.

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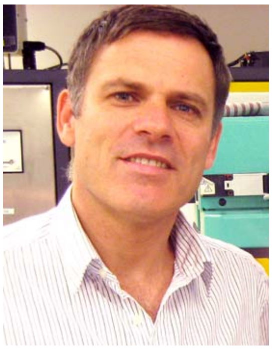

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